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Article

A comprehensive review of Value at Risk methodologies☆

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ABSTRACT

In this article we present a theoretical review of the existing literature on Value at Risk (VaR) specifically focussing on the development of new approaches for its estimation. We effect a deep analysis of the State of the Art, from standard approaches for measuring VaR to the more evolved, while highlighting their relative strengths and weaknesses. We will also review the backtesting procedures used to evaluate VaR approach performance. From a practical perspective, empirical literature shows that approaches based on the Extreme Value Theory and the Filtered Historical Simulation are the best methods for forecasting VaR. The Parametric method under skewed and fat-tail distributions also provides promising results especially when the assumption that standardised returns are independent and identically distributed is set aside and when time variations are considered in conditional high-order moments. Lastly, it appears that some asymmetric extensions of the CaViaR method provide results that are also promising.

1. Introduction

Basel I, also called the Basel Accord, is the agreement reached in 1988 in Basel (Switzerland) by the Basel Committee on Bank Supervision (BCBS), involving the chairmen of the central banks of Germany, Belgium, Canada, France, Italy, Japan, Luxembourg, Netherlands, Spain, Sweden, Switzerland, the United Kingdom and the United States of America. This accord provides recommendations on banking regulations with regard to credit, market and operational risks. Its purpose is to ensure that financial institutions hold enough capital on account to meet obligations and absorb unexpected losses.

For a financial institution measuring the risk it faces is an essential task. In the specific case of market risk, a possible method of measurement is the evaluation of losses likely to be incurred when the price of the portfolio assets falls. This is what Value at Risk (VaR) does. The portfolio VaR represents the maximum amount an investor may lose over a given time period with a given probability. Since the BCBS at the Bank for International Settlements requires a financial institution to meet capital requirements on the basis of VaR estimates, allowing them to use internal models for VaR calculations, this measurement has become a basic market risk management tool for financial institutions.[2] Consequently, it is not surprising that the last decade has witnessed the growth of academic literature comparing alternative modelling approaches and proposing new models for VaR estimations in an attempt to improve upon those already in existence.

Although the VaR concept is very simple, its calculation is not easy. The methodologies initially developed to calculate a portfolio VaR are (i) the variance–covariance approach, also called the Parametric method, (ii) the Historical Simulation (Non-parametric method) and (iii) the Monte Carlo simulation, which is a Semi-parametric method. As is well known, all these methodologies, usually called standard models, have numerous shortcomings, which have led to the development of new proposals (see Jorion, 2001).

Among Parametric approaches, the first model for VaR estimation is Riskmetrics, from Morgan (1996). The major drawback of this

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[2] When the Basel I Accord was concluded in 1988, no capital requirement was defined for the market risk. However, regulators soon recognised the risk to a banking system if insufficient capital was held to absorb the large sudden losses from huge exposures in capital markets. During the mid-90s, proposals were tabled for an amendment to the 1988 accord, requiring additional capital over and above the minimum required for credit risk. Finally, a market risk capital adequacy framework was adopted in 1995 for implementation in 1998. The 1995 Basel I Accord amendment provided a menu of approaches for determining the market risk capital requirements.

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model is the normal distribution assumption for financial returns. Empirical evidence shows that financial returns do not follow a normal distribution. The second relates to the model used to estimate financial return conditional volatility. The third involves the assumption that return is independent and identically distributed (iid). There is substantial empirical evidence to demonstrate that standardised financial returns distribution is not iid.

Given these drawbacks research on the Parametric method has moved in several directions. The first involves finding a more sophisticated volatility model capturing the characteristics observed in financial returns volatility. The second line of research involves searching for other density functions that capture skewness and kurtosis of financial returns. Finally, the third line of research considers that higher-order conditional moments are time-varying.

In the context of the Non-parametric method, several Non-parametric density estimation methods have been implemented, with improvement on the results obtained by Historical Simulation. In the framework of the Semi-parametric method, new approaches have been proposed: (i) the Filtered Historical Simulation, proposed by Barone-Adesi et al. (1999); (ii) the CaViaR method, proposed by Engle and Manganelli (2004) and (iii) the conditional and unconditional approaches based on the Extreme Value Theory. In this article, we will review the full range of methodologies developed to estimate VaR, from standard models to those recently proposed. We will expose the relative strengths and weaknesses of these methodologies, from both theoretical and practical perspectives. The article's objective is to provide the financial risk researcher with all the models and proposed developments for VaR estimation, bringing him to the limits of knowledge in this field.

The paper is structured as follows. In the next section, we review a full range of methodologies developed to estimate VaR. In Section 2.1, a non-parametric approach is presented. Parametric approaches are offered in Section 2.2, and semi-parametric approaches in Section 2.3. In Section 3, the procedures for measuring VaR adequacy are described and in Section 4, the empirical results obtained by papers dedicated to comparing VaR methodologies are shown. In Section 5, some important topics of VaR are discussed. The last section presents the main conclusions.

2. Value at Risk methods

According to Jorion (2001), "VaR measure is defined as the worst expected loss over a given horizon under normal market conditions at a given level of confidence. For instance, a bank might say that the daily VaR of its trading portfolio is $1 million at the 99 percent confidence level. In other words, under normal market conditions, only one percent of the time, the daily loss will exceed $1 million." In fact the VaR just indicates the most we can expect to lose if no negative event occurs.

The VaR is thus a conditional quantile of the asset return loss distribution. Among the main advantages of VaR are simplicity, wide applicability and universality (see Jorion, 1990, 1997).[3] Let r_1, r_2, r_3,. . ., r_n be identically distributed independent random variables representing the financial returns. Use $F(r)$ to denote the cumulative distribution function, $F(r) = \Pr(r < r|\Omega_{t-1})$ conditionally on the information set Ω_{t-1} that is available at time $t-1$. Assume that $\{r_t\}$ follows the stochastic process:

$$\begin{aligned} r_t &= \mu + \varepsilon_t \\ \varepsilon_t &= z_t\sigma_t \quad z_t \sim \text{iid}(0,1) \end{aligned} \tag{1}$$

where $\sigma_t^2 = E(z_t^2|\Omega_{t-1})$ and z_t has the conditional distribution function $G(z)$, $G(z) = Pr(z_t < z|\Omega_{t-1})$. The VaR with a given probability $\alpha \in (0, 1)$, denoted by VaR(α), is defined as the α quantile of the probability distribution of financial returns: $F(\text{VaR}(\alpha)) = Pr(r_t < \text{VaR}(\alpha)) = \alpha$ or $\text{VaR}(\alpha) = \inf\{v|P(r_t \le v) = \alpha\}$.

This quantile can be estimated in two different ways: (1) inverting the distribution function of financial returns, $F(r)$, and (2) inverting the distribution function of innovations, with regard to $G(z)$ the latter, it is also necessary to estimate σ_t^2.

$$\text{VaR}(\alpha) = F^{-1}(\alpha) = \mu + \sigma_t G^{-1}(\alpha) \tag{2}$$

Hence, a VaR model involves the specifications of $F(r)$ or $G(z)$. The estimation of these functions can be carried out using the following methods: (1) non-parametric methods; (2) parametric methods and (3) semi-parametric methods. Below we will describe the methodologies, which have been developed in each of these three cases to estimate VaR.[4]

2.1. Non-parametric methods

The Non-parametric approaches seek to measure a portfolio VaR without making strong assumptions about returns distribution. The essence of these approaches is to let data speak for themselves as much as possible and to use recent returns empirical distribution – not some assumed theoretical distribution – to estimate VaR.

All Non-parametric approaches are based on the underlying assumption that the near future will be sufficiently similar to the recent past for us to be able to use the data from the recent past to forecast the risk in the near future.

The Non-parametric approaches include (a) Historical Simulation and (b) Non-parametric density estimation methods.

2.1.1. Historical simulation

Historical Simulation is the most widely implemented Non-parametric approach. This method uses the empirical distribution of financial returns as an approximation for $F(r)$, thus VaR(α) is the α quantile of empirical distribution. To calculate the empirical distribution of financial returns, different sizes of samples can be considered.

The advantages and disadvantages of the Historical Simulation have been well documented by Down (2002). The two main advantages are as follows: (1) the method is very easy to implement, and (2) as this approach does not depend on parametric assumptions on the distribution of the return portfolio, it can accommodate wide tails, skewness and any other non-normal features in financial observations. The biggest potential weakness of this approach is that its results are completely dependent on the data set. If our data period is unusually quiet, Historical Simulation will often underestimate risk and if our data period is unusually volatile, Historical Simulation will often overestimate it. In addition, Historical Simulation approaches are sometimes slow to reflect major events, such as the increases in risk associated with sudden market turbulence.

The first papers involving the comparison of VaR methodologies, such as those by Beder (1995, 1996), Hendricks (1996), and Pritsker (1997), reported that the Historical Simulation performed at least as well as the methodologies developed in the early years,

[3] There is another market risk measurement, called Expected Shortfall (ES). ES measures the expected value of our losses if we get a loss in excess of VaR. So that, this measure tells us what to expect in a bad estate, while the VaR tells us nothing more than to expect a loss higher than the VaR itself. In Section 5, we will formally define this measure besides presenting some criticisms of VaR measurement.

[4] For a more pedagogic review of some of these methodologies (see Feria Domínguez, 2005).