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A model based on Copula Theory for sustainable and social responsible investments

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ABSTRACT

In this paper, a model is proposed that allows us to obtain a portfolio made up of sustainable and socially responsible (SR) investment funds. This portfolio tracks the one that investors might have chosen if they had not taken into account social, ethical and ecological (SEE) issues in their investment decisions. Therefore, in the first stage, reference portfolio exclusively made up of conventional funds is obtained. For the construction of the conventional portfolio the Prospect Theory has been used: net profits as the financial objective and error function as the utility function. In the second stage, a portfolio consisting exclusively of SR-funds is built. To do so, the reference portfolio is used as an ideal point, with the objectives of the SR-investor being the relative wealth with respect to the reference portfolio and the SEE quality of the portfolio. The relative wealth will be manipulated by a downside-risk measure, the Conditional Value at Risk (*CVaR*), and the periodic values of the portfolio. The second objective is the SR Quality of the portfolio, taking into account the personal values of a particular investor. This is built using Fuzzy Set Theory tools. We are faced with a multi-objective problem which is solved by using Goal Programming methodology. The estimation of both conventional and SR markets has been carried out by a semi-parametric approach by using the Copula Theory for modeling the dependence structure of the assets' returns. The approach has been applied to a set of 38 conventional and 12 ethical funds domiciled in Spain.

Un modelo basado en la teoría de cópulas para la inversión sostenible y socialmente responsable

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RESUMEN

El modelo que se propone permite obtener una cartera formada por fondos de inversión sostenibles y socialmente responsables (SR). Esta cartera sigue (tracking) a la cartera que hubiese elegido el inversor si no tuviera en cuenta valores sociales, éticos y ecológicos (SEE) en sus decisiones de inversión. Por tanto, la primera etapa del modelo consiste en obtener la cartera de referencia formada exclusivamente por fondos convencionales. Para la construcción de esta cartera se utiliza la Teoría de la Prospección: beneficios netos como objetivo financiero y la función de error como función de utilidad. En la segunda etapa, se construye una cartera formada exclusivamente fondos SR. Para ello, se utiliza la cartera de referencia como un punto ideal siendo los objetivos del inversor socialmente responsable la riqueza relativa respecto de la cartera de referencia y la calidad SEE de la cartera. Por tanto, se tiene un objetivo aleatorio –la riqueza relativa– que será manipulado mediante su Valor en Riesgo Condicional (*CVaR*) que es una medida de riesgo inferior, es decir, tiene en cuenta las peores pérdidas de la cartera, y los valores periódicos de la cartera. El segundo objetivo está referido a la calidad SR de la cartera atendiendo las

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creencias y valores personales de un inversor específico; se construye aplicando herramientas de la teoría de subconjuntos borrosos. Nos enfrentamos a un problema multi-objetivo que se resuelve mediante la Programación por Metas. La estimación de los mercados, convencional y SR, ha sido llevada a cabo mediante un enfoque semi-paramétrico utilizando la Teoría de Cópulas para modelar la estructura de dependencia de las rentabilidades de los activos. El modelo se ha implementado utilizando como mercado 38 fondos de inversión convencionales y 12 fondos éticos españoles.

1. Introduction

In a portfolio selection process, an investor seeks the combination of assets that best meets her needs in conditions of uncertainty. Therefore, in order to determine the optimum portfolio the investor needs to model, estimate, evaluate and manage such uncertainty. Under certain hypotheses, it is possible to estimate the parameters of the market which are necessary for the model, and afterwards to solve the optimization problem. In general, the investor focuses on a function of the portfolio value at the end of the holding period. The return and the profit-and-loss of the portfolio are examples of such functions.

The most popular approach for the selection of portfolios is the mean-variance model suggested by Markowitz (1952, 1959), where the investor tries to maximize her expected return for a fixed level of variance and within a set of investment constraints.

Modern Portfolio Theory (MPT) considers that investors can benefit from diversification by investing in financial assets with lower correlations. In MPT it is assumed that the financial returns follow a multivariate normal distribution. Therefore, the dependence between financial returns is described by the linear correlation coefficient and efficient portfolios are given by the traditional mean variance optimization program. However the variance has been criticized because it is a symmetric measure and treats downside risk and upside risk in the same way while the investors assign greater importance to downside risk than to upside risk (Ang, Chen, & Xing, 2006; Post & van Vliet, 2004).

Recently, there is a growing interest for the use of risk measures which taking into account the worst losses. Value at Risk (*VaR*)[1] and Conditional Value at Risk (*CVaR*)[2] have emerged as pertinent risk measures in finance and have found supporters in the financial community (Kaminski, Czupryna, & Szapiro, 2009; Krokhmal, Palmquist, & Uryasev, 2002; Meucci, 2007). The Basel Committee on Banking Supervision (Basel Committee on Banking Supervision, 2001, 2011) validated these new tools and enforced its use among financial institutions to monitor the riskiness of their investment policies. In addition, *VaR* has been criticized for not being a coherent risk measure (for more details, see Artzner, Delbaen, Eber, & Heath, 1999; Gilboa, 2009; McNeil, Frey, & Embrechts, 2005). For this reason, we use the *CVaR* as downside risk measure instead of the *VaR*.

For solving the portfolio selection problem, it is necessary to carry out three tasks. Firstly, to model the market, that is, to determine a random model to generate the asset prices at the investment horizon. In order to address this point we have used Copula Theory. It was introduced by Sklar (1959) but was not applied in finance until 1999 (for a deep description of the mathematical and statistical foundations of copulas, see Joe, 1997; Nelsen, 1999, and for a host of empirical applications of them, see Cherubini, Luciano, & Vecchiato, 2004; Jondeau, Poon, & Rockinger, 2007; McNeil et al., 2005). Sklar's theorem states that any multivariate distribution can be factored into the marginal cumulative distributions and a copula function describing the dependence between the components. An important advantage of copula lies in separating marginal distributions and dependence structure from joint distribution. Another good feature of copula functions is that they allow for different degrees of tail dependence: the upper tail dependence exists when there is a positive probability of positive outliers occurring jointly while the lower tail dependence is a negative probability of negative outliers occurring jointly. In particular, we consider *t*-copulas for their easy handling for simulation.

Secondly, to define optimality taking into consideration the investor's profile. This question reveals the difficulty in interpreting optimality under uncertainty. For this, it is necessary to use suitable surrogates on the real line of the investor's objectives, called "risk measures" in the context of cost or loss. Then, the question of which properties should verify the risk measures appears. These properties are known in the literature as "risk measure coherence" (Artzner et al., 1999). Lastly, to calculate an optimum portfolio, that is, to determine exactly or with a good approximation a portfolio which best fits the investor's profile. In most situations exact solutions are unattainable because analytical solutions do not exist due to numerical problems associated with the non-convexity of the problem; therefore, it is necessary to appeal to approximations.

In recent years business increasingly has been viewed as a major cause of social, environmental and economic problems. The instances of corporate scams and scandals have made that the investors bear in mind the quality of governance of companies and responsibility of their conduct (Gupta, Mehlawat, & Saxena, 2013). In this way, the investors are becoming conscious of the desirability of sustainable and responsible evaluation of the financial assets. Sustainable and Responsible Investing (SRI) refers to an investment process that, along with the traditional financial analysis, integrates the analysis of a company's social responsibility in pursuit of enhanced long-term returns. SRI provides a comprehensive way to assess a company's real value by including both corporate responsibility and sustainability measures in a company's valuation (EUROSIF, 2012).

This investment approach attracts an important number of people who wish their investments to be in accordance with their morality. SRI includes one or more of the following strategies in the search for, selection and follow-up of a portfolio: the exclusion of companies or "harmful" sectors (such as armaments, tobacco, alcohol etc.) to society (*negative screening*), positive filters in order to identify companies who enjoy a good reputation for issues such as the environmental protection, fair labor practices, quality of products, and worthy relationships regarding their presence in other countries (*positive screening*), investment in *best in class* companies in environmental and social performance, and communication and interaction with companies about their environmental, social and corporate governance actions by means of the exercise of political rights inherent in the ownership of enterprises (*constructive engagement* or *ethical overlay*) (Renneboog, Ter Horst, & Zhang,

[1] $VaR_\alpha(X)$ is defined as the maximum expected loss on an investment over a specified horizon at a given confidence level α

$$VaR_\alpha(X) = \sup\{x/P[X \geq x] > \alpha\}.$$

[2] $CVaR_\alpha(X)$ at confidence level α is defined as the conditional expected loss under the condition that it exceeds the $VaR_\alpha(X)$: $CVaR_\alpha(X) = E[X/X \geq VaR_\alpha(X)]$.