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Modeling simultaneous initiation and propagation of multiple hydraulic fractures under subcritical conditions

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ABSTRACT

Modeling the simultaneous propagation of multiple hydraulic fractures is important for stimulation of horizontal oil and gas wells. A new numerical model for the initiation and subsequent propagation of multiple hydraulic fractures allows the fractures to propagate under subcritical conditions. Including subcritical growth enables demonstration of the importance of time-dependent hydraulic fracture initiation and its impact on the subsequent growth of multiple hydraulic fractures, indicating that this phenomenon can strongly impact the behavior of simultaneously-growing hydraulic fractures, including reducing the suppression of some fractures during growth and reducing the pressure required to sustain fracture propagation.

1. Introduction

Multi-stage hydraulic fracturing (HF) is an essential technology for the completion of horizontal wells in unconventional hydrocarbon reservoirs. In engineering the design of multi-stage HF treatments of horizontal well stimulation, it is ideal to promote simultaneous growth of the fractures in all clusters per stage in order to reduce the number of non-producing perforation clusters [1,2]. In contrast to the ideal outcome of uniform fracture growth, laboratory experiments with multiple hydraulic fractures from one perforation interval indicate that interaction among the hydraulic fractures results in nonuniform growth with one fracture growing longer than the others and eventually dominating and taking all the fluid [3]. These so-called "stress shadow" effects have also been evidenced in field data [4] and predicted by numerical simulators [5–8].

Predicting the propagation of multiple fractures is a complex task since it hinges on the interplay among various factors, including the stress interaction among the fractures (the stress shadowing), the partitioning of the influx to each fracture, and the coupled fluid flow with elastic deformation in the cracks. Motivated by this challenging and important phenomenon, in recent years, efforts have been made to study the physical mechanisms governing the multiple HF growth, including the interaction among multiple fractures [9], the energetically advantageous partitioning of fluid among multiple hydraulic fractures [10,11], and the influence of the viscosity of the injection fluid and the pumping rate [12]. Furthermore, numerical models have been developed for solving the fully coupled problem of simultaneous growth of multiple fractures [13–15]. These invariably show a tendency for some hydraulic fractures to be suppressed due to stress interaction with other fractures. More insights regarding to the importance of perforation friction and separation between different fractures are given in these studies.

While increased attention has been given to studies of multiple HF growth, subcritical crack growth [16,17] is not typically considered as a factor affecting multiple HF growth. However, subcritical crack growth is shown to play a pivotal role in the initiation of hydraulic fracture(s) [18–21]. Laboratory experiments on granite, sandstone and limestone [18,21–23] show that by maintaining a constant subcritical fluid pressure, which is smaller than its critical value required to induce an instantaneous fracture initiation, HF growth can be achieved after a certain period of time. These experimental results indicate a correlation between this time to breakdown and the wellbore pressure. Furthermore, it is suggested that the underlying mechanism that governs the HF breakdown in such a delayed manner is due to the stable crack propagation under subcritical wellbore conditions, i.e., subcritical crack growth. The classical theory of Linear Elastic Fracture Mechanics

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(LEFM) does not allow fracture initiation when the mode I (opening) stress intensity factor, $K_{\rm I}$, is less than the fracture toughness (or critical stress intensity factor), $K_{\rm IC}$. However, according to Atkinson [17,24], fractures can propagate stably under stresses that are insufficient to satisfy such a condition (i.e., when $K_{\rm I} < K_{\rm IC}$), with velocities several orders of magnitude smaller than the LEFM propagation velocity. Such crack growth is governed by an empirical power law, referred to as the subcritical crack growth law [16,17], for describing the relation between the fracture tip velocity, V, and the stress intensity factor, $K_{\rm I}$, that is

$$V = A\left(\frac{K_{\rm I}}{K_{\rm IC}}\right)^n \tag{1}$$

where n is the subcritical crack growth index, and A is a constant characteristic velocity typically taken as an upper bound on the crack propagation speed when $K_I \to K_{\rm IC}$. Using the subcritical crack growth law (Eq. (1)), numerical simulations have previously been carried out on the initiation and following propagation of a single hydraulic fracture by Lu et al. [20]. The results show that subcritical growth can lead to a reduced wellbore pressure and increased fracture length compared with the classical model.

Although it is seen that subcritical crack growth can have substantial effect on the single HF growth, its influence on the case of multiple fractures remains unknown. Bunger and Lu [19] proposed that the time-dependent HF initiation may be one of the fundamental phenomena for generating and growing multiple fractures simultaneously. Thus motivated, in this paper a fully coupled numerical model is presented for solving the initiation and subsequent propagation of an array of N axisymmetric hydraulic fractures that accounts for subcritical fracture growth. This model stems from the classical LEFM model and is integrated with the subcritical crack growth law (Eq. (1)). The problem formulation is given in Section 2. Section 3 presents the scaling of the governing equations, and details of the numerical algorithm are described in Section 4. Our numerical solution is validated in Section 5, and this model is further utilized to study the impact of the key factors. Finally, we draw the conclusions from this work in Section 6.

2. Problem formulation

2.1. Model description

We consider the problem of simultaneous initiation and propagation of N axisymmetric, transverse hydraulic fractures growing from a horizontal well with radius a (see Fig. 1) in an impermeable linearly elastic rock characterized by Young's modulus E, Poisson's ratio ν, and fracture toughness $K_{\rm IC}$. An incompressible Newtonian fluid with viscosity μ is

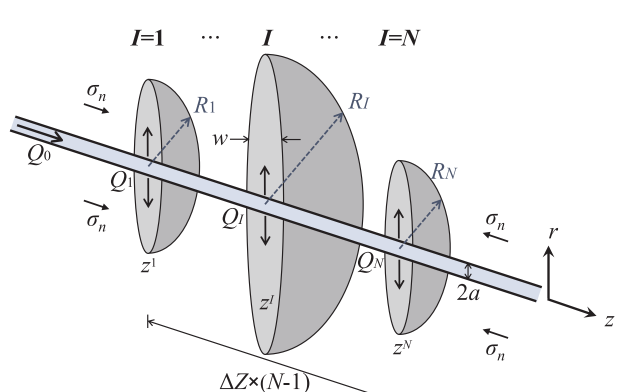

Fig. 1. Sketch of a single stage of multi-stage HF treatment for an array of N axisymmetric hydraulic fractures (clusters) growing simultaneously from a horizontal wellbore injected by a constant rate Q_0. The fractures are placed along the direction of the well with a constant spacing, ΔZ.

injected at a constant volumetric rate Q_0 into the wellbore system with a finite compressibility U. For each fracture $I = 1, ..., N$, an initial defect of radius R_0^I is assigned for modeling the initiation of the fracture. Introduction of a finite initial defect from the well and simulating how this defect begins to grow and how the resulting hydraulic fracture develops over the early stages of its life is a common approach to investigating hydraulic fracture ”initiation” [25–28]. Such an approach is substantially simplifying the entire complex phenomenon of crack nucleation, nevertheless, it can be useful as a way to gain insight about early stage hydraulic fracture growth from a tractable model.

The spacing, ΔZ, between the fractures in the array is held constant, and we assume that all fractures remain planar while they propagate from a horizontal well under the minimum in-situ stress, σ_n, acting orthogonal to the fracture plane. The interference between fractures can suppress propagation of some fractures, and also cause their propagation paths to curve, but prior studies [9,29] indicate that the magnitude of curving decreases with increasing in-situ deviatoric stress (i.e., the difference between the maximum and minimum horizontal in-situ stresses). Under a large enough deviatoric stress, which applies to most unconventional hydrocarbon reservoirs, the out-of-plane deflection may be negligible [29]. Thus, we neglect the possibility of fracture curving and path deflection as the surrounding stress fields evolve, and restrict the hydraulic fractures to grow in a circular geometry in a planar, parallel array.

It is also worth noting that we are here neglecting pressure drop at the entry of the hydraulic fractures, for example due to flow through the perforations connecting the wellbore to the formation. High entry pressure loss can significantly affect multiple hydraulic fracture growth, promoting more uniform growth (e.g. Lecampion and Desroches [14]). Since our goal in this work is to study the impact of subcritical crack growth on the initiation and propagation of multiple hydraulic fractures, we believe it is useful to assume zero friction loss in all fractures. Putting in large perforation friction could overwhelm the impact that we are trying to investigate. Furthermore, fluid leak-off to the rock, and the fluid lag (separation between the fracture tip and the fluid front) are also assumed to be negligible.

As depicted in Fig. 1, the wellbore is drilled along the direction of minimum in-situ stress σ_n (z direction), and r is the radial coordinate along the direction of crack propagation, with $r = 0$ corresponding to the center of the wellbore. Consequently, we have $r \in (a, R^I)$ for the Ith crack. The fractures are placed uniformly along the wellbore such that $z^I = (I{-}1)\Delta Z$ for $I = 1, ..., N$. Finally, the solution for this problem consists of the fracture radius $R^I(t)$, the crack width $w^I(r, t)$, the wellbore pressure $p_w(t)$ (i.e., the fluid pressure along the welbore wall $p_f(a, t)$), and the fluid pressure $p_f^I(r, t)$ in each fracture I.

2.2. Governing equations

2.2.1. Elasticity

Following Lecampion and Desroches [14], the distributed dislocation theory [30] is used to describe the relation of the normal (D_{zz}) and shear (D_{rz}) ring dislocation dipoles with the net stress acting on each of the fractures. For the Ith fracture, the elasticity equation is given in the form of two boundary integral equations for normal and shear net loading along the crack, which are dependent on the fluid pressure p_f, the in-situ stress σ_n, the normal and shear interaction stresses between fractures σ_{int}^I and τ_{int}^I, as well as the near wellbore effect. The boundary integral equations are given by

$$\begin{aligned} &p_f(r, z^I) - \sigma_n(r, z^I) - \sigma_{int}^I(r, z^I) = \\ &\int_a^{R^I} [\sigma_{zzzz}(r, z^I; r', z^I) w(r', z^I) + \sigma_{zzrz}(r, z^I; r', z^I) v(r', z^I)] dr' \\ &\tau(r, z^I) = -\tau_{int}^I(r, z^I) = \\ &\int_a^{R^I} [\sigma_{rzzz}(r, z^I; r', z^I) w(r', z^I) + \sigma_{rzrz}(r, z^I; r', z^I) v(r', z^I)] dr' \end{aligned} \tag{2}$$

where $w = -D_{zz}$ is the fracture width, and v represents the shear displacement