Computers and Geotechnics 104 (2018) 207–215

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

# A shear hardening plasticity model with nonlinear shear strength criterion for municipal solid waste

Xilin Lü[a,*], Dawei Xue[a], Maosong Huang[a], Keng-Wit Lim[b]

[a] Key Laboratory of Geotechnical and Underground Engineering of Ministry of Education, Department of Geotechnical Engineering, Tongji University, Shanghai 200092, China

[b] Engineering Mechanics Division, Simpson Gumpertz & Heger, Los Angeles, CA 90015, USA

ARTICLE INFO

*Keywords:*
Municipal solid waste
Backward Euler implicit integration
Shear hardening plasticity
Bearing capacity

ABSTRACT

A shear hardening plasticity model with nonlinear strength criterion and appropriate stress-dilatancy description is presented to numerically study the bearing behavior of landfills. Simulations of drained triaxial tests are performed to illustrate the capability of the model in capturing the mechanical behavior of municipal solid waste. The bearing response and failure mode of a landfill foundation are determined using 2D and 3D finite element simulations. In contrast to the general shear failure mode obtained from rigid plasticity, the failure obtained from the hardening model shows a local mode, and prior to failure, a larger reaction force is produced for the same settlement.

## 1. Introduction

Landfill is commonly used for the disposal of municipal solid waste (MSW) in developed and developing countries. MSW is often vertically stacked on a hillside or vertically piled up at flat areas to form landfill sites. Due to stringent government regulations and growing environmental concerns, it is generally difficult to find new sites for landfills. As a result, existing landfill sites are often required to store more MSWs, beyond their original intended capacity. Additionally, with the rapid development and population growth in cities, facilities may be constructed on landfill sites. Under increasing loads, the landfill sites will possibly undergo instability. It is a key design issue to gain an understanding into the deformation behavior and bearing capacity of landfill sites.

MSW is known to be a complex material with various constituent types [28,37]. Therefore, the bearing behavior of a landfill site will be different from that of a conventional foundation. MSW in China typically consists of metal and glass, plastic, dust, organic material, and fiber contents including paper, wood, textiles, and leather [5]. Due to this heterogeneity and composition complexity, the peak shear strength of MSW generally exhibits a nonlinear behavior with respect to mean stress [26]. In previous studies, however, this nonlinearity is often ignored and the Mohr-Coulomb strength criterion is used to characterize the shear strength of MSW [8,10,16,20]. As a result, this linear criterion may result in large errors in the predicted shear strength of MSW [22]. In view of the disadvantage of the Mohr-Coulomb model, nonlinear strength criteria have been proposed to better describe the shear strength [2,32,35,36,43]. A constitutive model that can reasonably capture the main physical and mechanical properties of MSW [9,15,17,24] should be proposed. One of the important properties pertains to the volumetric response of MSW. In most existing models, the plastic volumetric strain is used as a state variable, which increases monotonically upon loading. Therefore, only volumetric contraction can be captured. In reality, however, when large, compressible particles and fibrous material that initially serve as reinforcement in MSW are degraded into other weaker substances due to aging [8] or mechanical and biological treatment [3], the MSW may also exhibit dilatant behavior. A shear hardening model in which an appropriate stress-dilatancy mechanism is incorporated can simulate well the dilatant behavior of mechanical and biological treated MSW [24]. Models that consider the effect of degradation on the physical and mechanical properties of MSW [10] and the compressibility of particles [27] can also accurately reflect the physical and mechanical properties of MSW, but are generally complex in terms of numerical implementation.

Most of the previous constitutive modeling studies are mainly confined to the simulation of elemental behavior, and do not involve general boundary value problems. The stability of landfill slopes has received a lot of attention since the last century [5,30] and it becomes increasingly important to be able to model actual field conditions. For landfills that are located on a hillside, an inclined slope is the main

* Corresponding author at: Key Laboratory of Geotechnical and Underground Engineering of Ministry of Education, Department of Geotechnical Engineering, Tongji University, Shanghai 200092, China.

*E-mail address:* xilinlu@tongji.edu.cn (X. Lü).

https://doi.org/10.1016/j.compgeo.2018.08.019
Received 3 May 2018; Received in revised form 15 July 2018; Accepted 28 August 2018
0266-352X/ 

topographic feature of a landfill; therefore, slope stability is an important issue. But for flat areas, the stability issues are mainly related to foundation bearing capacity. The bearing capacity problems may be caused by the addition of MSW or construction of facilities on landfill sites. Previous studies on the bearing capacity analysis are mainly based on finite element simulations with an elastic perfectly plastic model. Literature shows that some MSW, especially fresh MSW, behaves similar to reinforced soil [4,17,18,34,39,41,42]. According to the experimental study by Huang and Tatsuoka [14], the failure mode of reinforced soil is close to a local shear failure. When reinforcement material exists (e.g. discrete fibers), strain hardening behavior of soil is significant [39]. These studies, however, have not considered the effect of strain hardening on bearing behavior when carrying out numerical studies [1,6,12,19,25,29,45]. Although the bearing behavior of MSW has not been directly studied using a representative constitutive relationship, similar analyses using conventional models have been attempted. For example, Hegde et al. [13] analyzed the ultimate bearing properties of reinforced soil using a modified Cam-clay model, which, however, was developed based on a linear strength criterion. Therefore, these analysis results still cannot be directly applied to the bearing capacity analysis of landfill.

This paper presents the formulation and application of an elasto-plastic MSW model [24], in which a shear hardening rule in a nonlinear yield function is incorporated. The backward Euler implicit integration algorithm is used to update the stress, state parameters, and tangent stiffness matrix. Drained triaxial tests are numerically simulated to illustrate the capability of the model in estimating the stress-strain relationship and volumetric behavior of MSW. Through two- and three-dimensional (2D and 3D, respectively) bearing capacity simulations, the response of the MSW model proposed in this paper is compared to that of the rigid plasticity model.

## 2. Shear hardening model for municipal solid waste

In this section, the formulation and stress integration procedure for the shear hardening model with nonlinear shear strength criterion established by Lu et al. [24] is proposed.

### 2.1. Formulation of the rate-form constitutive model

Conventional MSW models typically build their yield function on a state variable that is related to the plastic volumetric strain $\varepsilon_v^p$ [2,32,35,36,43]. The plastic volumetric strain produced by shearing monotonically increases upon loading, so that only volumetric contraction can be captured. To reflect the dilatant response of some MSW (e.g., old age MSW [3], mechanical and biological treated MSW [8]), a shear hardening mechanism combined with a stress-dilatancy relationship is used [24].

The yield function of the model is

$$F = q + Mp^{\xi} = 0 \tag{1}$$

where $p = \sigma_{ii}/3$ is the mean stress; $p_0$ is the initial mean stress; $q = \sqrt{3s_{ij}s_{ij}/2}$ is the equivalent shear stress; $s_{ij} = \sigma_{ij} - \delta_{ij}p$ is the deviatoric stress; $\xi$ is the strength parameter and for $\xi = 1$, a linear strength model is recovered; $M$ is the hardening function.

The adopted hardening function obeys a shear hardening rule [23,33]

$$d\varepsilon_s^p = \frac{-pM}{h_s G(M_f - M)} dM \tag{2}$$

where $G$ is the elastic shear modulus; $M_f$ defines the condition at critical state; $\varepsilon_s^p = \sqrt{2e_{ij}e_{ij}/3}$ is equivalent plastic shear strain, and $e_{ij} = \varepsilon_{ij} - \varepsilon_{kk}\delta_{ij}/3$; $d\varepsilon_s^p$ is the incremental equivalent plastic shear strain; $h_s$ is a fitting parameter.

The plastic potential is

$$Q = q - \frac{AM_c p}{1-A}\left[1 - \left(\frac{p}{p_0}\right)^{-(1-A)}\right] = 0 \tag{3}$$

where $A$ is a material constant that can be obtained by fitting the experimental curve of volumetric strain and axial strain, and an increase in $A$ leads to a more volumetric dilatancy response; $M_c$ is a stress-dilatancy parameter that corresponds to the transition of volumetric behavior from compressive to dilative.

Integrating Eq. (3), the following equation can be obtained

$$\varepsilon_s^p = \frac{p}{h_s G}\left[M + M_f \ln\left(1 - \frac{M}{M_f}\right)\right] \tag{4}$$

Eq. (4) is the shear hardening rule for this model.

The bulk elastic modulus $K$ and shear modulus $G$ are obtained by the following functions [24].

$$\begin{cases} K = \frac{2(1+v)}{3(1-2v)} G \\ G = G_0 p_{at} \frac{(e^* - e_0)^2}{1+e_0} \sqrt{\frac{p'}{p_{at}}} \end{cases} \tag{5}$$

where $G_0$ is the initial shear modulus; $e_o$ is the initial void ratio, and it can be can be obtained from the conversion between basic physical parameters, i.e. the volume and weight (the detailed procedure is desribed in [11]); $e^*$ is a regression constant and can be determined by fitting a loading-unloading test; $p_{at}$ is the atmospheric pressure.

### 2.2. Numerical implementation

Using the backward Euler implicit integration algorithm, the stress increment, state parameters, and the tangent stiffness matrix of the corresponding residual equations are updated at each loading step. The detailed backward Euler implicit integration algorithm is shown in Appendix A. In numerical modeling, the unknown variables are solved using the following residual equations

$$\boldsymbol{R}(\boldsymbol{x}_{n+1}) = 0 \tag{6}$$

where $\boldsymbol{x}_{n+1}$ represent mean stress $p$, generalized shear stress $q$, plastic operator $\lambda$ and state parameter $M$ at incremental step n + 1.

Using the return mapping approach and the Newton-Raphson method, Eq. (6) are converted to

$${}^{k+1}\boldsymbol{x}_{n+1} = {}^{k}\boldsymbol{x}_{n+1} - {}^{k}\{\boldsymbol{D}\}^{-1}\boldsymbol{R}({}^{k}\boldsymbol{x}_{n+1}) = 0 \tag{7}$$

where $k$ denotes the iterative step, ${}^{k}\{\boldsymbol{D}\}^{-1} = \left\{\frac{\boldsymbol{R}({}^{k}\boldsymbol{x})}{\boldsymbol{x}}\right\}$. The unknown variables can be solved by iteration.

The stress tensor then can be obtained as

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{s}_{n+1} + \delta \cdot p \tag{8}$$

where $\boldsymbol{s}$ denotes the deviatoric stress, $\delta$ is the Kronecker delta.

Taking the derivative of Eq. (6) with respect to $\Delta\boldsymbol{\varepsilon}$, we can obtain

$$\frac{\partial \boldsymbol{R}(\boldsymbol{x}_{n+1})}{\partial \Delta\boldsymbol{\varepsilon}} = 0 \tag{9}$$

and according to Appendix B, tangent matrix can be obtained as

$$\boldsymbol{C} = \frac{\partial \Delta\boldsymbol{\sigma}}{\partial \Delta\boldsymbol{\varepsilon}} \tag{10}$$

## 3. Validation by drained triaxial tests

Experimental data from previous studies [3,38] is used to verify the applicability of the proposed MSW model in simulating stress-strain relations and volumetric characteristics. As shown in Fig. 1, the finite element mesh is composed of 700 eight-noded brick elements. The bottom of the specimen is fully fixed. After a confining stress is applied, vertical displacement is applied at the specimen top.