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Parameterized model based blind intrinsic chirp source separation

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ABSTRACT

The blind source separation (BSS) concerns recovering sources from their mixtures. Specifically, the sources in this paper, named intrinsic chirp sources (ICSs), are modeled as the linear combination of non-linear chirp components (NCCs). A novel method is developed here to address the blind separation issue of them. Firstly, all the mixtures at each channel are decomposed into a series of NCCs by a parameterized decomposition approach. It can adapt to NCCs with time-frequency (T-F) distribution suffering from bad T-F concentration and non-disjoint T-F overlapping. Next, the reconstructed NCCs can be clustered into corresponding ICS according to the fact that the NCCs belonging to the identical ICS share the same column in mixing matrix. The source recovery and mixing matrix estimation are finally accomplished based on the clustering consequence. Three simulations demonstrate the capability of our method in dealing with challenging under-determined BSS cases and its potential in practical applications.

1. Introduction

Blind source separation (BSS) focuses on recovering multiple sources from a set of mixing observations without *a priori* knowledge of mixing system and sources. It can find great applications in many signal-concerned engineering occasions, such as audio and speech signal processing [1–3], radar signal processing [4,5], machine fault diagnosis [6], and biomedical signal processing [7,8]. Three classical mathematical models [6] are usually adopted to approximate the mixing system, i.e., instantaneous mixing model, anechoic mixing model and echoic mixing model. For the tractability of mixing model in mathematics, some *a priori* assumptions are usually compensated for the sources in specific application situations, such as statistical independence between sources [9–11], sparsity of sources under certain dictionary [12] and special distribution of sources [13]. However, few efforts have been tried for instantaneous amplitude (IA) and instantaneous frequency (IF) modulated chirp source. Named as intrinsic chirp source (ICS) in this paper, it possesses wide applications [14–16].

The BSS approaches, based on spatial time-frequency distribution (STFD) [17–19], have been proposed one after another and attracted plenty of attention in recent two decades. They are able to separate various kinds of sources with sparse time-frequency (T-F) structure, in which ICS is also included. Equipped with a proper T-F point selection scheme, one of the approaches [20] based on STFD can solve the blind separation problem of ICSs with IF overlapping in over-determined case, where the number of sensors is more than that of sources. However, the over-determined case is not always valid in practice. A cluster-based T-F under-determined BSS (TF-UBSS) approach [21] has been developed later for under-determined case, where there are more sources than sensors. This approach, assisted with a subspace-based algorithm [19,22], is also able to separate the ICSs with non-disjoint T-F overlapping. Though the STFD-based BSS approaches [5] can address the blind ICS separation issue in not a few occasions, they will still encounter difficulties under more challenging circumstances. For example, they should undertake a significant computational burden when heaps of T-F overlapping points arise. And they are incapable of dealing with the situation where there exist more sources than sensors at T-F overlapping region with the unique representation property unsatisfied [23]. Besides the STFD-based approaches, there also grew some other kinds of BSS approaches [4,24,25] that can resolve the blind ICS separation problem in certain cases, but they will all suffer in aforementioned challenging scenarios.

In this paper, a specialized blind ICS separation method is put forward to overcome the difficulties mentioned above for complex-formed instantaneous mixing model. The reason of selecting instantaneous mixing model is that it satisfies the requirements of the practical ICS applications [4,5]. The ICS, in our method, is firstly defined by a linear combination of non-linear chirp components (NCCs), whose IAs and IFs are parameterized by complex redun-

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dant Fourier bases [26] and polynomial bases, respectively. Such a model can adapt to wide variety of chirp sources. And it is more general than those given in Ref. [4], where IA is assumed as a constant and ICS is merely modeled by one NCC.

In previous blind separation approaches [5,17–21] for ICSs, the ICSs are usually recovered by transforming the clustered auto-source T-F values back into time-domain. Different from that, the NCCs are regarded as clustering objects in our method and the ICSs are recovered by adding up all the NCCs clustered into the same category. To be specific, the method proposed in this paper firstly decomposes the observations at all channels into a series of NCCs by a parameterized decomposition approach. It is immune to the NCCs with complex T-F characters, such as non-disjoint T-F overlapping and above-mentioned challenging scenarios suffered by STFD-based approaches. Thanks to the parameterized NCC model, the IAs and IFs of each component can also be obtained immediately after decomposition. The next significant step of our blind ICS separation method is the clustering of reconstructed NCCs. The decomposition consequence indicates that NCCs belonging to the same ICS correspond to the identical column vector in mixing matrix [25]. This characteristic is used here to cluster NCCs into corresponding ICSs depending on the distances between their corresponding column vectors, which can be estimated from the IAs of reconstructed NCCs. An empirical distance threshold is adopted here for clustering, which is robust to the noise strength below certain level. The ICSs recovery and mixing matrix estimation are finally accomplished with the clustering result.

The remainder of this paper is organized as follows. In Section 2, the blind ICS separation problem is formulated under the frame of general BSS mathematical model. A novel blind ICS separation method is proposed in Section 3, which is illustrated through two parts, i.e., observation decomposition and NCC clustering. And finally, Section 4 demonstrates the proposed method by three examples, which is followed by the conclusion section, i.e., Section 5.

2. Problem formulation

The complex-formed instantaneous mixing model of BSS issue at certain channel m can be mathematically expressed as

$$x_m(t) = \sum_{n=1}^{N} a_{mn} s_n(t) + n_m(t), \tag{1}$$

where $s_n(t)$ $(n = 1, 2, \cdots, N)$ is the nth source signal; $x_m(t)$ $(m = 1, 2, \cdots, M)$ is the mth observation; $n_m(t)$ is the white Gaussian noise at mth observation; $a_{mn} \in \mathbb{C}$ quantifies the amplitude attenuation rate $|a_{mn}|$ and phase delay $\angle a_{mn}$ of $s_n(t)$ at mth channel. The corresponding matrix mode of Eq. (1) is

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{2}$$

where

$$\begin{aligned}
&\mathbf{x}(t) = \left[x_1(t), x_2(t), \cdots, x_M(t)\right]^T, \\
&\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_N], \quad \mathbf{a}_n = [a_{1n}, a_{2n}, \cdots, a_{Mn}]^T \in \mathbb{C}^M, \\
&\mathbf{s}(t) = \left[s_1(t), s_2(t), \cdots, s_N(t)\right]^T, \\
&\mathbf{n}(t) = \left[n_1(t), n_2(t), \cdots, n_M(t)\right]^T, \\
&E\left[\mathbf{n}(t)\right] = \mathbf{0}_{M\times 1}, \qquad E\left[\mathbf{n}(t)\mathbf{n}^{\boldsymbol{T}}(t)\right] = \operatorname{diag}\left(\sigma_1^2, \cdots, \sigma_M^2\right).
\end{aligned}$$

Herein, $E[\cdot]$ denotes mathematical expectation and σ_m^2 $(m = 1, 2, \cdots, M)$ is the variance of white Gaussian noise $n_m(t)$. The scaling ambiguity of mixing matrix $\mathbf{A}$ is eliminated by assuming $\|\mathbf{a}_n\|_2 = 1$, and that $\mathbf{a}_\kappa \neq \mu \mathbf{a}_\iota$ is held for arbitrary nonzero constant μ and unequal integral pair $\kappa, \iota \in \{1, 2, \cdots, N\}$.

In an important application scene of BSS, radar signal processing, the mixing matrix $\mathbf{A}$ in instantaneous mixing model is usually modeled as [24]

$$\mathbf{A} = \mathbf{A}(\boldsymbol{\Theta}) = \left[\mathbf{a}_1(\Theta_1), \mathbf{a}_2(\Theta_2), \cdots, \mathbf{a}_N(\Theta_N)\right], \tag{3}$$

where $\mathbf{a}_n(\Theta_n)$ is named as steering vector, which just depends on the direction of arrival (DOA) Θ_n if the array pattern is predetermined. For general case in Eq. (2), there are $M - 1$ unknown variables in column vector $\mathbf{a}_n$ due to $\|\mathbf{a}_n\|_2 = 1$.

In this paper, the sources in Eq. (2) are multi-component IA and IF modulated chirp signals, which are named as intrinsic chirp sources (ICSs) and mathematically defined as follows

Definition 1. As a continuous function, the source in Eq. (1) $s_n(t)$: $\mathbb{R} \to \mathbb{R}$, $s_n(t) \in L^\infty(\mathbb{R})$ is named as the ICS, if

$$s_n(t) = \sum_{k=1}^{K_n} c_n^{(k)}(t), \tag{4}$$

where the non-linear chirp component (NCC) $c_n^{(k)}(t)$ is formulated as [27]

$$c_n^{(k)}(t) = A_n^{(k)}(t) \exp\left[j\left(2\pi \int_0^t f_n^{(k)}(\tau)d\tau + \varphi_n^{(k)}\right)\right], \tag{5}$$

with $A_n^{(k)}(t)$ and $f_n^{(k)}(t)$ satisfying the following restrictions

$$\begin{aligned}
&A_n^{(k)}(t) \in C^1(\mathbb{R}) \cap L^\infty(\mathbb{R}), \quad \inf A_n^{(k)}(t) > 0, \quad \sup A_n^{(k)}(t) < \infty, \\
&f_n^{(k)}(t) \in C^1(\mathbb{R}), \quad \inf f_n^{(k)}(t) > 0, \quad \sup f_n^{(k)}(t) < \infty, \\
&\left|\frac{dA_n^{(k)}(t)}{dt}\right|, \left|\frac{df_n^{(k)}(t)}{dt}\right| \le \epsilon \left|f_n^{(k)}(t)\right|.
\end{aligned}$$

Herein, $A_n^{(k)}(t)$ and $f_n^{(k)}(t)$ represent modulated IA and IF of NCC $c_n^{(k)}(t)$; $\varphi_n^{(k)} \in [0, 2\pi]$ is the initial phase of $c_n^{(k)}(t)$ at $t = 0$; the constant ϵ controls the modulation strength; $K_n \in \mathbb{N}_+$ in Eq. (4) denotes number of NCCs included in source $s_n(t)$. In addition, it should be noted here that all the superscripts of mathematical notations with brackets denote the sequence numbers in this paper while those without brackets represent the powers.

The two primary missions in BSS are source recovery and mixing matrix estimation. Specifically, in blind ICS separation problem, the source recovery mission can be accomplished by NCC reconstruction and clustering.

3. Blind ICS separation method

3.1. Main idea

In this section, a blind intrinsic chirp source (ICS) separation method is proposed to achieve the source recovery and mixing matrix estimation. This method is executed in two steps. Firstly, the observations in all the channels are decomposed in units of non-linear chirp components (NCCs) by a parameterized decomposition approach. The decomposed observation at mth channel can be rewritten as

$$\begin{aligned}
x_m(t) = {} & a_{m1} \sum_{k=1}^{K_1} c_1^{(k)}(t) + a_{m2} \sum_{k=1}^{K_2} c_2^{(k)}(t) + \cdots \\
& + a_{mN} \sum_{k=1}^{K_N} c_N^{(k)}(t) + n_m(t).
\end{aligned} \tag{6}$$