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Numerical and experimental assessment of random matrix theory to quantify uncertainty in aerospace structures

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ABSTRACT

The increased complexities of modernized structures have catalyzed additional research over the past few decades to accurately quantify the variabilities within static designs, which can be categorized into parametric, non-parametric and experimental uncertainties. The majority of works have dealt with parametric uncertainty alone and utilize algorithms that require *priori* knowledge of the input uncertainty. This is problematic since there is a large number of uncertain variables that typically percolate physical engineering problems which are difficult to characterize using precise probability distributions. A unified method to encompass these stochastic properties in a computationally efficient manner is needed and one such approach is using random matrix theory (RMT), which has integrated and validated its importance through various fields, however largely excluding the aerospace sector. This study utilizes RMT and the Wishart distribution to characterize modal uncertainties that arise due to parametric, non-parametric and experimental variabilities. Numerical studies are undertaken with finite element models of a rectangular wing and an aircraft wing box to validate RMT and the Marchenko-Pastur (MP) density function. An experimental study is also conducted on six nominally designed aircraft T-tails, where RMT is used to provide a quantitative bound on the unknown uncertainties that exist within the physical system.

1. Introduction

The need for an efficient method to conduct Uncertainty Quantification (UQ) has become a focal area of research due to the ubiquitous nature of uncertainties within complex engineering systems. The importance of UQ stems from improving cost and time effectiveness within various design stages and providing engineers additional platforms to refine their approaches. Over the past decade, a plethora of studies have been conducted across a number of fields, such as nuclear engineering [1], climate change modelling [2], the agriculture industry [3] and structural dynamics [4]. The latter is of particular interest since inadequate knowledge of the structures around us can place people at risk and lead to significant maintenance costs. As such, a fundamental practice within the engineering design process involves introducing conservative safety factor margins for a structure through the over-design of its material properties for a range of critical cases [5]. One such example within the aviation field is the 15% flutter margin requirement for all US military aircraft, which was set by the Federal Aviation Authority (FAA) in the mid 1960s and persists currently even with the numerous advances in computational methods made since then [6].

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This leads to an underlying issue which permeates structural analysis: finding a credible numerical tool which can capture the effects of uncertainty. In the aerospace sector, finite element codes are quintessential for analyzing high order degree-of-freedom (DOF) structures, however they inherently assume a linear static analysis with completely deterministic baseline system parameters [7]. This means parametric uncertainty, non-parametric uncertainty (model uncertainty) and experimental errors are not accounted for[4]. There has been a focus on dealing with parametric uncertainty in existing literature, however quantifying other sources of error that percolate in model design, or experimental studies, such as human error in the construction of a structure, has not received as much attention.

To improve the robustness of numerical models, fundamental frameworks such as the stochastic finite element method (SFEM) [8] have been continually developed where the uncertainties concerning geometric parameters and material properties (i.e. Young's modulus, Poisson's ratio, material sizing) can be characterized by parametric approaches. Adikhari [9] summarizes the three main approaches as perturbation based, projection based and simulation using Monte Carlo. The first category involves expanding the perturbation using Taylor series or a more sophisticated method such as Neumann expansion [10], however both tend to suffer if the uncertainty is non-Gaussian [9]. The next category is projection based where the solution vector is "projected" on to a complete stochastic basis. The Polynomial Chaos Expansion (PCE) is one such implementation which has been shown to be suitable to quantify uncertainty in aerospace problems. Examples include [11,12] where it was applied to an aircraft T-tail and to a composite flat plate respectively.

Uncertainty due to modelling error has also received attention due to its inherent difficulty, since the contributing variables are not available as a *priori*. These errors can be associated with, but not limited to, non-linearities in the equation of motion, damping and modelling of structural joints [4]. Since no explicit system parameters can be used to propagate these effects, the typical approach to deal with model uncertainties is the so-called non-parametric method proposed by Soize [13], which utilized random matrix theory (RMT). RMT was introduced by Wigner in 1951 [14] to formulate a statistical model of the resonances observed in the scattering of neutrons across an element's nuclei. Since then it has been utilized in a plethora of fields ranging from number theory [15] to wireless communication [16]. Adoption and refinement of RMT in structural mechanics, seen in references [17–20], has justified the use of Wishart Matrices [21] when dealing with stochastic damped n-degree-of-freedom linear dynamic systems. Recent applications of this formulation focus on analyzing stochastic frequency response function (FRF) characteristics and determining changes in modal behaviour. Two examples are Birklaryan et al. [22], who characterized uncertainty in pier dynamics and Vishwanathan et al. [23] who applied RMT to quantify shifts in natural frequency in 2D wings. Adhikari et al. [24] extended the above formulation to derive an analytical eigenvalue density for a random undamped eigenvalue problem. In particular, they concluded that the Marchenko-Pastur (MP) density function [25] provided a reasonable approximation of the analytical and experimental limiting eigenvalue density for stochastic systems of varying complexity.

Despite the aforementioned advancements in the field of RMT, it has seen sparse usage within the aerospace sector, where mechanical models and physical structures have a large degree of complexity. This study will serve as an overview and assessment of RMT and the non-parametric method to characterize both numerical and experimental uncertainty in aerospace structures. Three primary uses of RMT are investigated: predicting shifts in modal behaviour, modelling stochastic eigenvalue density using the MP density function and providing a quantitative bound on the unknown errors that percolate in experimental studies. FE models of a simple rectangular wing and a complete aircraft wingbox are examined for their eigenvalue characteristics. These test cases are provided to highlight the versatility of RMT in being able to deal with uncertain material/geometric properties. It should be noted however that the main benefit of RMT over other UQ methods is in dealing with non-parametric problems, where there is no *priori* knowledge of the input uncertainty. This is showcased via the experimental study, which is conducted on six nominally designed aircraft T-tails constructed in the school of Aeronautical, Mechanical and Mechatronic Engineering (AMME) at the University of Sydney. Modal testing is performed for each T-tail and compared to a numerical model with known sources of parametric variability controlled for. The differences are analyzed, leaving only the non-parametric and experimental errors remaining. This paper therefore adds to the limited number of UQ experiments seen in literature [4,26–28] and to the best of this author's knowledge is the first to apply RMT to a realistic environment found in aerospace engineering.

The outline of the paper is as follows; an overview of RMT, including the problem formulation, is outlined in Section 2; the numerical FE studies are conducted in Section 3; the experimental T-tail study is found in Section 4; and finally a summary of the paper and key contributions are provided in Section 5.

2. RMT background

2.1. Problem formulation

A stochastic n-degree-of-freedom linear dynamic system can be expressed as in Eqs. (1) and (2) below:

$$\mathbf{M}\ddot{q}(t) + \mathbf{C}\dot{q}(t) + \mathbf{K}q(t) = \mathbf{f}(t) \tag{1}$$

$$\mathbf{H}(\omega) = [-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}]^{-1} \tag{2}$$

where $\mathbf{f}(t)$ is the forcing vector and $\mathbf{q}(t)$ is the response vector. Matrices $\mathbf{M}, \mathbf{C}$ and $\mathbf{K}$ represent the mass, damping and stiffness respectively and are random matrices. $\mathbf{H}(\omega)$ is the frequency response function, where ω is the input frequency. This