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Viscoelastic shear deformable microplates: Nonlinear forced resonant characteristics

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ABSTRACT

A viscoelastic model for a shear-deformable microplate is developed in this paper while accounting for geometric nonlinearities. Nonlinear numerical solutions are conducted to examine the resonant oscillations of the microsystem. The geometrically nonlinear theoretical model is developed utilising the Kelvin-Voigt viscoelastic model, to account for nonlinear dissipation, the modified version of the couple-stress theory, to account for small-scale characteristics, and the third-order deformation theory, to account for shear stress. The constitutive relations for both the classical and higher-order stress tensors are constructed and are divided into elastic and viscous components. The elastic components are used to develop the potential strain energy and the viscous components are employed to model the virtual work of damping (energy dissipation). Additionally, the microplate motion energy is developed while accounting for all in-plane, out-of-plane, and rotational motions. A distributed harmonic load, as the representative an external force, is applied to the microsystem and the corresponding virtual work is obtained. The generalised Hamilton's principle is applied to the virtual works and variations of the energy terms, resulting in the nonlinear equations of motion of the microsystem. Being of partial differential type, the equations of motion are discretised into a set of nonlinearly coupled ordinary differential equations consisting of nonlinear geometric and nonlinear damping terms. A solution procedure for the forced oscillation analysis of the microsystem is developed using a continuation method. Different diagrams are constructed to examine the nonlinear resonant characteristics of the viscoelastic shear deformable microplate and to highlight the nonlinear dependency of the Kelvin-Voigt viscoelastic damping mechanism on the oscillation amplitude, for a geometrically nonlinear model.

1. Introduction

The bending and oscillations of microscale beams and plates are commonly used to measure external parameters in different microelectromechanical systems (MEMS) [1–4] such as microscale sensors, resonators, energy harvesters, and accelerometers [5–16]. It is shown that microresonators display nonlinear energy dissipation and that the internal energy dissipation plays an important role in the forced or free motions of microscale structures. In the presence of geometric nonlinearities, the material damping mechanisms such as Kelvin-Voigt, Zener, and Maxwell, become nonlinearly

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amplitude-dependent and hence are capable of capturing the damping nonlinearities associated with large-amplitude oscillations. In this paper, Kevin-Voigt material damping is utilised to highlight the effect of damping nonlinearities in the nonlinear forced resonant oscillations of a shear deformable microplate. Furthermore, it has been discovered experimentally [17–19] that when a structure becomes small, the size affects the motion/bending behaviour of the system which is characterised by an additional stiffness in a linear sense. An advanced continuum mechanics theory should be employed to capture the size effects [20,21]; this paper utilises the modified couple stress (MCS) theory [22–27].

The number of studies on the *elastic* models of microplates incorporating linear damping mechanism is quite large, which includes both geometrically linear and nonlinear studies. For instance, using the modified strain-gradient theory, Ashoori et al. [28] obtained the linear out-of-plane equation of motion of a microscale plate. Hashemi and Samaei [29] employed the nonlocal Mindlin plate theory to conduct a linear buckling investigation on micro/nonoplates under in-plane forces. Further investigation was conducted by Wang et al. [30], who employed the strain gradient theory in conjunction with the Kirchhoff plate theory, so as to study the linear size-dependent behaviour of microplates. Jomehzadeh et al. [31] conduced a linear vibration analysis on a microplate making use of the MCS theory. The investigations were continued by Nabian et al. [32], who analysed the stability of a functionally graded microplate under electrostatic and hydrostatic pressure. Employing a meshless method together with MCS theory, Roque et al. [33] examined the bending characteristics of a shear deformable microplate. Based on the nonlocal Eringen theory, Farajpour et al. [34] investigated the linear buckling response of a graphene plate. Apart from the linear studies in the literature, there are also several studies of microplates utilising geometrically nonlinear models. For instance, employing the MCS theory, Asghari [35] developed the geometrically nonlinear size-dependent equations of motion of a microplate. Thai and Choi [36] developed size-dependent nonlinear functionally graded Kirchhoff and Mindlin plate models on the basis of the MCS theory.

All the aforementioned valuable studies analysed the linear/nonlinear behaviour of microplates through use of elastic models with linear damping mechanism. The present study analyses for the first time the *forced nonlinear resonant oscillations of a shear deformable viscoelastic microplate* while accounting for *stiffness* and *damping* nonlinearities. The nonlinear equations of motion of the microplate are derived through use of the (i) MCS theory, (ii) the third-order shear deformable plate theory, (iii) the Kelvin-Voigt viscoelastic material damping model, and (iv) generalised Hamilton's principle. Furthermore, von-Kármán strain-displacement nonlinearities are accounted for, which due to employment of a Kelvin-Voigt material damping model, results in both geometric and damping nonlinearities. Though use of a double-dimensional Galerkin technique and via incorporating basis functions consistent with the fully clamped boundary conditions, the partial differential equations of motion are reduced and transformed into equations of ordinary differential type. Extensive numerical calculations are then conduced employing a continuation technique. The nonlinear resonant characteristics of the viscoelastic shear deformable microplate are examined while highlighting the nonlinear dependency of the damping to oscillation amplitude.

2. Model development for a viscoelastic microplate

A geometrically nonlinear model of the third-order shear deformable microplate is developed in this section taking into account two different damping mechanisms, i.e. the linear viscous damping mechanism and the Kelvin-Voigt viscoelastic internal damping mechanism which consists of linear and nonlinear parts due to presence of geometric nonlinearities. The reason for the presence of two damping mechanisms is to be able to compare them in the numerical simulations. The considered microplate is shown in Fig. 1 within a Cartesian coordinate system (x,y,z). The microplate's dimensions in the x and y directions are denoted by a and b, respectively, while its dimension in the z direction (i.e. thickness) is shown by h. The microplate is under a distributed excitation load of $F_1\cos(\omega t)$ with F_1 and ω being the forcing amplitude and frequency, respectively, and t being time. The employed third-order shear deformation theory contains five independent variables, i.e. three displacements and two rotations. The mid-plane displacement components are denoted by u, v, and w, in the x, y, and z directions, respectively. The rotations of the transverse normal at $z = 0$ are shown by ϕ_1 and ϕ_2. Furthermore, the components of the displacement vector are denoted by u_x, u_y, and u_z.

In what follows, the third-order sear deformation theory, the modified couple stress theory, the Kelvin-Voigt viscoelastic model, and the generalised Hamilton's principle are employed to develop a discretised nonlinear model for the microplate.

Taking into account the von Kármán strain nonlinearities, the strain tensor components for a third-order shear deformable microplate can be written as [37]

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \end{Bmatrix} + \begin{Bmatrix} \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\ \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \\ \frac{1}{2}\left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right) \end{Bmatrix} + z\begin{Bmatrix} \frac{\partial \phi_1}{\partial x} \\ \frac{\partial \phi_2}{\partial y} \\ \frac{1}{2}\left(\frac{\partial \phi_1}{\partial y}+\frac{\partial \phi_2}{\partial x}\right) \end{Bmatrix} - \frac{4z^3}{3h^2}\begin{Bmatrix} \frac{\partial \phi_1}{\partial x}+\frac{\partial^2 w}{\partial x^2} \\ \frac{\partial \phi_2}{\partial y}+\frac{\partial^2 w}{\partial y^2} \\ \frac{1}{2}\left(\frac{\partial \phi_1}{\partial y}+\frac{\partial \phi_2}{\partial x}+2\frac{\partial^2 w}{\partial x \partial y}\right) \end{Bmatrix}, \tag{1}$$

$$\begin{Bmatrix} \varepsilon_{xz} \\ \varepsilon_{yz} \end{Bmatrix} = \frac{1}{2}\left(1-\frac{4z^2}{h^2}\right)\begin{Bmatrix} \phi_1+\frac{\partial w}{\partial x} \\ \phi_2+\frac{\partial w}{\partial y} \end{Bmatrix}. \tag{2}$$