Signal Processing 154 (2019) 148–158

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Team-optimal online estimation of dynamic parameters over distributed tree networks

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ARTICLE INFO

Article history:
Received 19 December 2017
Revised 24 May 2018
Accepted 11 August 2018
Available online 22 August 2018

Keywords:
Optimal estimation
Distributed network
Tree networks
Dynamic parameter
Online estimation

ABSTRACT

We study online parameter estimation over a distributed network, where the nodes in the network collaboratively estimate a dynamically evolving parameter using noisy observations. The nodes in the network are equipped with processing and communication capabilities and can share their observations or local estimates with their neighbors. The conventional distributed estimation algorithms cannot perform the team-optimal online estimation in the finite horizon global mean-square error sense (MSE). To this end, we present a team-optimal distributed estimation algorithm through the disclosure of local estimates for tracking an underlying dynamic parameter. We first show that the optimal estimation can be achieved through the diffusion of all the time stamped observations for any arbitrary network and prove that the team optimality through disclosure of local estimates is only possible for certain network topologies such as tree networks. We then derive an iterative algorithm to recursively calculate the combination weights of the disclosed information and construct the team-optimal estimate for each time step. Through series of simulations, we demonstrate the superior performance of the proposed algorithm with respect to the state-of-the-art diffusion distributed estimation algorithms regarding the convergence rate and the finite horizon MSE levels. We also show that while conventional distributed estimation schemes cannot track highly dynamic parameters, through optimal weight and estimate construction, the proposed algorithm presents a stable MSE performance.

1. Introduction

1.1. Preliminaries

Recently, due to advancements in information technologies, distributed learning and estimation techniques have attracted significant attention thanks to their fast convergence and robustness properties for fast streaming data [1–7]. In a distributed estimation framework, we consider a network of agents observing a temporal signal about an underlying state, possibly coming from different spatial sources with different statistics. Each agent in the network is equipped with communication and processing capabilities. The aim of each agent is to estimate the underlying parameter of interest, as an example, by minimizing the expected Euclidean distance between the estimate and the true value of the state (the minimum mean-square estimation (MMSE)). The agents in the network are connected to a set of neighboring nodes and can exchange information, i.e. observations and/or estimates, between them to improve their learning process. To illustrate, assume a network of emission sensors distributed over a greenhouse to monitor the CO_2 levels for a precision agriculture application [8]. Since the agents would collect different observations from different parts of the area, they can cooperate in the network to rapidly learn and track the true CO_2 levels for an enhanced intervention.

In this regard, the distributed learning and estimation has been extensively studied in the signal processing and machine learning literatures [9–17]. However, the classical methods either do not consider the information diffusion scheme among the agents and/or construction of the optimal combination methods to obtain the MMSE performance or are not applicable for real-time applications [13]. To this end, in this paper, we present an approach to obtain the optimal distributed online estimation in a team framework by exploiting the network structure and the information disclosure and combination when the underlying state is non-stationary and time varying. In this framework, agents in our network cooperate

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https://doi.org/10.1016/j.sigpro.2018.08.007
0165-1684/

with their neighbors as a "team" to minimize a predefined team cost based on the actions of each agent. To achieve this aim, each agent generates a local estimate for the underlying dynamic system parameter and then constructs certain messages to share with its neighbors at each time instance. Based on the sharing process, each agent's goal is to obtain a solution that minimizes the predefined team cost, i.e., the "team-optimal" solution.

1.2. Related work

There exists an extensive research on distributed estimation of a time invariant or a dynamic state parameter, which are mainly studied under centralized and decentralized distributed learning frameworks [9–16,18,19]. In the centralized frameworks, all the agents in the network are connected to a fusion center and each agent transmits its information to the center for the construction of the final estimate [9,18,19]. Since all the information is collected by a single node such methods do not require any specific information sharing scheme and constructing the global optimal estimate is straightforward. However, this approach has serious disadvantages regarding communication and computation loads on the network, i.e. transmitting all the peripheral information to a single node requires a huge communication bandwidth and processing all the collected information on a single unit requires a significant computational power [9,12].

In the alternative decentralized frameworks, each agent in the network has a different set of neighboring nodes consisting of spatially close ones and exchange information only with these nodes to overcome the former problems [20]. In these approaches, agents only disclose (or share) their local information on the underlying parameter and combine the received information to produce their final estimates. In this framework, the information efficiently propagates through the network to improve the overall performance [21].

In the consensus approach of the decentralized frameworks, all the agents in the network reach to a "consensus" on their estimates after collecting and processing their information locally [15,16]. However, this approach either requires a use of two time scales to reach to the consensus immediately or decaying learning rate for constructing the consensus among the agents in time [16,22]. The use of two time scales limits the performance of the network on real-time applications. On the other hand, the use of decaying learning rates hinders the ability of the system to adaptively adjust or learn in time varying environments [13].

In [14–16,23] and [24], authors present diffusion based approaches for distributed estimation, where the network is able to respond to the fast-streaming data in an online manner by using a single time scale. In the diffusion based strategies, agents process their observations locally and disclose the corresponding estimates to the neighboring nodes and improve their performance through combining the received estimates. In [13], authors prove that the diffusion based approaches outperform single time scale consensus strategies regarding the global MSE performance. However, neither of these methods consider the network topology or information disclosure procedures to obtain a globally optimal solution. On the other hand, in [10,12], diffusion incremental solutions are shown to reach to the optimal estimate by defining a certain path through the network, which is not practical against the fast streaming data or the dynamic configurations.

In [25], authors presented a novel approach to obtain the team-optimal distributed estimation of a static underlying parameter by exploiting the network structure, and the optimal information disclosure and combination without any incremental path requirements. However, in most of the real-life applications, the underlying parameter is subject to a change, i.e. it evolves in time [26]. Although there exists different studies on the distributed estimation of a dynamic parameter, these algorithms again do not consider the correlation of the disclosed information between the agents in the network due to the dynamic evolution of the underlying parameter [3,26–28]. Hence, these algorithms cannot achieve the team-optimal estimation and the problem requires a different approach than the solutions available in the literature.

To this end, we work on the team-optimal estimation of dynamic parameters over distributed networks. We first use the framework of Sayin et al. [29] to establish the model and the problem. Then, we introduce the efficient and optimal distributed learning (EODL) algorithm for the online estimation of dynamic parameters and prove that it is only applicable over certain network topologies. We also show the superior performance of the proposed method compared to the state-of-the-art methods through numerical examples.

1.3. Main contributions

We list our main contributions as follows:

- We show that the team optimal estimation is possible over any arbitrary network if the agents disclose the time and node stamped versions of their observations even under dynamic environments.
- We prove that the team-optimality is possible only over certain network topologies, e.g., tree networks, if the agents only disclose their local estimates.
- We introduce an algorithm for estimating a dynamic parameter that achieves the team-optimal error lower bound over these certain networks.
- We derive an efficient information sharing and combination scheme to reduce the communication load over such networks.
- We provide numerical examples to illustrate the convergence and the steady-state performance improvements achieved by our algorithm with respect to the state-of-the-art methods.

We organize the paper as follows. In Section 2, we present the team framework for the dynamic parameter estimation and show that the optimal estimate can be constructed through diffusion of the time stamped information. Then, in Section 3, we prove that the team-optimal estimation through disclosure of local estimates can be achieved only under certain network topologies. Later in Section 4, we provide an iterative algorithm to construct the optimal combination weights and the estimate over such networks. We demonstrate the performance of the proposed algorithm through series of simulations in Section 5 and conclude the paper with final remarks in Section 6.

2. Team framework for distributed estimation

In this paper, all random variables are represented as uppercase calligraphic letters, i.e. $\mathcal{X}$, and all the realizations of these variables are presented as their lowercase characters, i.e. x. All the vectors are column vectors and denoted by boldface lowercase letters.

We consider a distributed network with m agents equipped with processing and communication capabilities. We form the network as an undirected graph, where vertices and edges represent the agents and the communication links respectively, as shown in Fig. 1. For each agent i, we denote the set of agents, whose information is available to the agent i after transmission over k communication links (after k-hops) as $\mathbf{N}_i^{(k)}$. We define $\mathbf{N}_i^{(k)}$ as

$$\mathbf{N}_i^{(k)} = \{j_{1^{(k)}}, \cdots, j_{\pi_i^{(k)}}\}, \tag{1}$$

where $\pi_i^{(k)} = |\mathbf{N}_i^{(k)}|$ is the cardinality of the set $\mathbf{N}_i^{(k)}$. We assume that $\mathbf{N}_i^{(0)} = \{i\}$ and $\mathbf{N}_i^{(k)} = \emptyset$ for $k < 0$. In Fig. 1, we demonstrate the first neighborhood of the agent i, where $\mathbf{N}_i = \{j_1, j_2, j_3\}$ and $\pi_i = 3$.