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Model predictive control of piezo-actuated structures using reduced order models☆

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ABSTRACT

A model predictive control method for a flexible structure consisting of two beams, equipped with macro fiber composite (MFC) patches as piezoelectric actuators is presented. Adjacent beams are connected at the tip by an elastic string. The equations of motion are derived by the evaluation of the extended Hamilton's principle. Spatially dependent parameters are considered, due to different material characteristics of the carrier beams and the MFC patches. The spatially dependent variable is approximated using Galerkin's method, leading to the state space representation of the beam structure by means of ordinary differential equations (ODEs). In order to avoid approximation errors, a high order approximation of the distributed-parameter system is required. With a model order reduction a small system describing approximately the same dynamics is obtained. A model predictive controller is designed, analyzed and evaluated in a simulation and at the experimental set-up. For the real-time capability of the model predictive controller a move blocking method by downsampling the prediction horizon is presented.

1. Introduction

Flexible structures with distributed sensors and actuators, that are bonded or embedded in an elastic carrier structure, are also called smart structures (Banks, Smith, & Wang, 1996; Clark, Saunders, & Gibbs, 1998; Janocha, 2007; Stanewsky, 2001) and arise in a wide range of applications. For example, in large earth based telescopes, the optical wave fronts coming from celestial objects are disturbed by atmospheric turbulence. Using adaptive optics the phase differences in the disturbed wave fronts are detected by wave front sensors in the telescopes and compensated by controlling the shape of deformable mirrors in real time (Hardy, 1998; Preumont, 2011).

Smart structures with piezoelectric sensors and actuators find also application in active vibration suppression (Fuller, Elliott, & Nelson, 1996). Examples include active vibration damping in the blades of helicopters (Giurgiutiu, 2000; Shaw & Albion, 1981) or in spacecraft structures (Won, Belvin, Sparks, & Sulla, 1994). Vibrations in mechanical structures may lead to undesired noise. Active vibration suppression can be designed in a way to reduce the noise inside an aircraft (Fuller, Snyder, Hansen, & Silcox, 1992; Grewal, Zimcik, Hurtubise, & Leigh, 2000) or inside a car (Hurlebaus & Gaul, 2006).

Moreover the shape control of smart structures, where the surface profile can be actively manipulated, has become increasingly popular (Janocha, 2007). For example in aerospace applications an adaptive wing can be hingelessly and smoothly deformed by using smart structures. Thereby, the aerodynamic performance, e.g., lift to drag ratio, maneuver capabilities, and aeroelastic effects are improved compared to the adaption of the shape profile using flaps (Antcliff & Mcgowan, 2000; Saggere & Kota, 1999; Yousefi-Koma & Zimcik, 2003). Many different control approaches for manipulating the shape profile have been applied to piezo-actuated flexible beam structures, as for example infinite-dimensional control concepts (Kugi & Thull, 2005; Kugi, Thull, & Kuhnen, 2006), passivity based control methods (Kugi & Schlacher, 2002), as well as H_2 and H_∞ control approaches (Kugi, 2001). Flatness based feedforward control has been applied with great success for shape control of flexible beam structures (Meurer, 2013; Rudolph & Woittennek, 2002; Schröck, Meurer, & Kugi, 2010a) and combined with feedback control to compensate occurring oscillations (Kater & Meurer, 2015). For vibration attenuation Lyapunov based control (Bailey & Hubbard, 1985), positive position feedback control (Fanson & Caughey, 1990), optimal vibration control (Hanagud, Obal, & Calisej, 1992), as well as fuzzy controller (De Abreu & Ribeiro, 2002; Lin & Liu, 2006) and back propagation neural network controller (Qiu, Zhang, & Ye, 2012) have been applied.

In this contribution, a model predictive controller is designed to perform a rest-to-rest motion of a piezo-actuated coupled beam structure,

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which is schematically shown in Fig. 1. Model predictive control (MPC) is a method which explicitly uses the mathematical model to predict the future dynamics of a system. The optimal input is calculated in each sampling step by solving an optimization problem, in which the error between system or output variables and a predetermined final value is minimized (Camacho & Bordons, 2007; Rawlings & Mayne, 2009). By imposing constraints on the optimization problem physical bounds of actuators are directly considered by the model predictive controller (Maciejowski, 2002; Wang, 2009). MPC is well studied for linear systems as well as for nonlinear systems (Allgöwer, Badgwell, Qin, Rawlings, & Wright, 1999; Allgöwer & Zheng, 2012; Findeisen & Allgöwer, 2002; Grüne & Pannek, 2011) and can be found in a large field of applications, e.g., process and chemical industries. An excellent review of MPC applications in industry appears in Qin, Badgwell, Qin, and Badgwell (2003).

In general, MPC is applied to systems, which are characterized by slow system dynamics so that the optimization problem can be solved within each sampling step in real-time. Contrary to this smart structures are typically characterized by fast oscillatory dynamics, which complicates the design and the implementation of model predictive controllers. Moreover, coupled flexible structures often show the clustering of weakly damped eigenmodes and require to consider high dimensional models to accurately reflect the system dynamics. To address these issues a benchmark example for the MPC of smart structures is subsequently considered in terms of an interconnected flexible beam structure with embedded piezoelectric macro fiber composite (MFC) patches as distributed actuators, see Fig. 1. The MFC patches are elastically deformed by exploiting the indirect piezoelectric effect, i.e., the conversion of an electrical voltage into a mechanical strain, which is transferred to the beam as bending moment.

The mathematical modeling of the beam structure leads to a distributed parameter description by means of partial differential equations (PDEs). In the analysis and control design for distributed parameter systems one distinguishes between early lumping and late lumping methods. In early lumping methods the spatial–temporal dynamics of the distributed parameter system is approximated by a finite dimensional system of ordinary differential equations (ODE), whereas in late lumping methods the complete dynamics of the distributed parameter system is taken into consideration. Therefore, approximation errors, e.g., spillover effects, may occur in the early lumping approach. However, the mathematical complexity in late lumping is significantly higher than in early lumping. In this contribution, an early lumping approach is applied. Approximation errors and spillover effects are addressed by using a high order approximation of the PDEs. The order and therefore the complexity of the resulting system is systematically reduced using a model order reduction method. A wide range of methods exists for model order reduction, e.g., modal truncation (Antoulas, 2005; Davison, 1966), proper orthogonal decomposition (Kerschen, Golinval, Vakakis, & Bergman, 2005), or Krylov subspace model order reduction methods (Daniel, Siong, & Chay, 2004; Hochbruck & Lubich, 1997; Saad, 1981). In this contribution an extension to classical balanced truncation (Antoulas, 2005; Laub, Heath, Paige, & Ward, 1987; Moore, 1981) is applied, in order to preserve the stability and the second order structure of the mechanical system (Reis & Stykel, 2008).

The application of an MPC method to a highly dynamical weakly damped system like the flexible beam structure of Fig. 1 is an interesting and rather young field of research, which has to cope with the challenges arising from the computational complexity for solving the optimization problem in real-time on dedicated real-time hardware. MPC for fast positioning of the end-effector of a single elastic arm has been successfully implemented in Bossi, Rottenbacher, Mimmi, and Magni (2011). Differing from these results for a boundary controlled single beam, in the following a flexible beam structure with coupled beams and piezoelectric in-domain actuation is considered and model order reduction is addressed to systematically achieve a model description of moderate size. Additional differences arise from the different system characteristics, which, e.g., require high sampling frequencies to resolve the system dynamics. The implications of this are also studied subsequently. The model predictive controller is designed based on the time-discrete model of the flexible beam structure and constraints are imposed on the input variables in order to consider physical boundaries of actuators. The static optimization problem is solved by using Hildreth's quadratic programming procedure (Wang, 2009). For the application to the experimental set-up, a high sampling frequency is required to capture the complete dynamics of the system. However, the resulting computational complexity of the optimization problem makes the model predictive controller not directly real-time capable. A common approach to deal with this problem is to reduce the degrees of freedom of the optimization problem by fixing the input variables over several time-steps (Cagienard, Grieder, Kerrigan, & Morari, 2007; Lee, Chikkula, Yu, & Kantor, 1995). A move blocking method by downsampling the prediction horizon is presented, whereby the model predictive controller can be executed in real-time with a sufficiently high sampling frequency.

The paper is organized as follows: in Section 2 the equations of motion are derived for the considered coupled flexible beam structure. Using a Galerkin approach a high order approximation of the distributed-parameter system is introduced, which leads to a good approximation quality but large computational times in view of MPC. Model order reduction is studied in Section 3 to determine a low-order model properly reflecting the dynamics of the high-order system that is, however, suitable for a real-time MPC realization. The model predictive controller is designed based on the reduced model in Section 4 and is evaluated at the experimental set-up in Section 5.

2. Energy based modeling

The equations of motion of the flexible beam structure are derived in a general manner for an arbitrary number of beams equipped with an arbitrary number of MFC patches by making use of the extended Hamilton's principle. By taking into account the weak formulation of the equations of motion we follow a Galerkin approach to determine a high-order finite-dimensional approximation of the governing PDE model.

2.1. Beam configuration

The considered flexible beam structure, schematically shown in Fig. 1, consists of n carbon fiber beams, which are clamped at $z_n^1 = 0\,\mathrm{m}$. At the free end $z_n^1 = \ell_n$ masses are attached. The masses of adjacent beams are connected by an elastic string, modeled as linear spring. All beams and MFC patches have the same dimensions. The spatially distributed MFC patches are located pairwise symmetrically on each side of the beam. Their spatial characteristics, schematically shown in Fig. 2, is given by

$$\Omega_{n,m}(z) = h\left(z - z_{n,m}^1\right) - h\left(z - z_{n,m}^1 - \ell_{n,m}\right) \tag{1}$$

where h denotes the Heaviside function, $\ell_{n,m}$ is the length and $z_{n,m}^1$ is the border of the mth MFC patch pair on the nth beam along the z-axis. By exploiting the indirect piezoelectric effect, an electrical voltage is converted into a mechanical strain, which is transferred to the beam as a bending moment proportional to the voltage, leading to a deflection of the beam. The used MFC patches are operated with an anti-symmetric voltage supply $U_{n,m}^{l/r}(t) \in [-500, 1000]$ V, where the superscript l/r denotes the left and the right MFC patch, respectively, and the doublesubscript n, m refers to the nth beam and the mth MFC patch pair on the nth beam. To achieve a symmetric input voltage supply of $U_{n,m}(t) \in [-1000, 1000]$ V a bias of $U_b = 500\,\mathrm{V}$ is added to the input voltage, such that the input voltage of each MFC patch pair is given by

$$U_{n,m}^{l/r}(t) = U_b \pm U_{n,m}(t) \tag{2}$$

with $n \in \{1, \dots, N\}$ and $m \in \{1, \dots, M_n\}$, where N and M_n are the number of carrier beams and the number of MFC patches on the nth beam,