Applied Mathematical Modelling 65 (2019) 464–488

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

# Modeling of progressive failures in quasi-brittle media based on a temporal stress-redistribution mechanism

Jin Xing Liu[a,*], Nai Gang Liang[b], Ai Kah Soh[c]

[a] *Faculty of Civil Engineering and Mechanics, Jiangsu University, Xuefu Road 301, Zhenjiang 212013, Jiangsu, China*
[b] *State key laboratory of Nonlinear Mechanics, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China*
[c] *School of Engineering, Monash University Malaysia, Jalan Lagoon Selatan, Bandar Sunway 47500, Selangor Darul Ehsan, Malaysia*

ARTICLE INFO

*Article history:*
Received 9 December 2017
Revised 22 July 2018
Accepted 23 August 2018
Available online 6 September 2018

*Keywords:*
Progressive quasi-brittle failure
Stress redistribution
Damage-induced softening
Viscous force field
Event-driven algorithm
Strain rate sensitivity

ABSTRACT

A new attempt is made to simulate progressive failure processes in heterogeneous brittle materials such as concrete, ceramics, rocks etc., by considering the time-dependence of stress redistributions induced by local breakages. Two mechanisms of stress redistribution are incorporated into the proposed model in order to account for the influence of each local breakage on the remaining specimen: (1) one is the immediate release of internal forces in the breaking element, which is assumed to happen within an infinitesimal time when compared with the characteristic time of external loadings. The release of such internal forces is hence suddenly applied to the remaining specimen, which is considered to take time to deform correspondingly due to material viscosity. This deformation delay is implemented by introducing a viscous force (VF) field prevailing in the entire specimen. (2) The other is the gradual release of previously stored VF fields, whose characteristic time is assumed to be material-dependent. Here the release of VF is approximated as stepwise for simplicity. The proposed model is found to be capable of overcoming the unreasonably-low-ductility problem encountered in many existing lattice models when it comes to the uniaxial tensile test. Furthermore, the force–displacement response obviously depends on the ratio of the VF releasing time to the characteristic time of external loading, showing trends agreeing with experimental observations. Compared with results without viscosity, the failure pattern is more scattering, and the force–displacement curve has a higher peak load and a more ductile post-peak tail.



## 1. Introduction

Quasi-brittle materials can be natural or artificial, including but not limited to concrete, cementitious composites, sea-ice, cohesive and frozen soils, and toughened ceramics. In above media, cracks propagate progressively under external loadings, which, compared with ideally brittle materials like glass, has some distinctive features such as post-peak softening, and has attracted intensive research interests (e.g. [1–6]). Lattice-type models have been widely used to study such phenomena [7–15], in which the continuum-like material was considered as a network composed of particular fundamental elements such as bars, beams or various link elements, usually based on the principle of strain energy equivalence [11,13], or by installing relevant springs between each two neighboring rigid bodies [16,17].

* Corresponding author.
*E-mail address:* jxliu@mails.ucas.ac.cn (J.X. Liu).

https://doi.org/10.1016/j.apm.2018.08.037

Failure happens when the deformation or stress at a particular material point violates the failure criterion. Tang [1] assumed that the maximum tensile strength is dominant in determining rock fracture and modeled progressive failures leading to collapse in rock and associated seismicity. van Mier et al. [9] detected local breakages by comparing the maximum allowable tensile stress with the present normal stress which was the combination of normal force and bending moment in the beam element. In the rigid-body-spring model, Bolander and Saito [16] adopted the Mohr–Coulomb criterion with a tensile cut-off, accounting for both normal and shear stresses. In the generalized beam (GB) lattice model proposed by Liu et al. [13], a failure rule similar to [16] was adopted to determine the elemental breakage, in conjunction with the consideration of separation/contact of cracked element surfaces. When a lattice serves as the micromechanics model of a material, the heterogeneous mesostructure is explicitly considered, naturally incorporating the relevant characteristic length. Thus, the explicit heterogeneity in the lattice is dominant, and the meshsize plays a secondary role when compared with homogenized macro-continuum models [18]. The crack band model proposed in [19] can be feasible and straightforward to account for such a size dependency.

In this study, we focus on the influence of ongoing local breakages to the remaining specimen. For this end, in the development of lattice-type models, there have been two main kinds of non-iterative algorithms, i.e. the load–unload method and the force–release method [17,20], which are based on two totally different assumptions in terms of characteristic time scales during progressive failures.

A post-peak softening response is generally exhibited when laboratory samples of a quasi-brittle material are loaded in tension or unconfined compression. Even though massive successful applications of lattice models in analyzing failure phenomena have been achieved, the macroscopic softening-related issue, i.e. that the load–displacement curve obtained by the lattice model shows a much lower ductility than experimental observations [20] particularly during the post-peak stage, calls for further investigations. Lilliu and van Mier [21] built a three-dimensional lattice model which was still not able to fully recur the realistic ductility. The macroscopic softening has been sometimes taken as material softening even though it is indeed a mixture of material and structural properties, resulting from the micro-cracking, rather than simply a fundamental response of the studied material [11,22–24]. van Mier [22] postulated that a lattice adopting an elastic-purely brittle fracture law at the level of the aggregates, interfaces and matrix is suitable for studying fracture mechanisms leading to global softening. Bai et al. [25], Krajcinovic and Rinaldi [26] and Li and Ren [27] emphasized that stochastic damage evolution plays an important role. Many other investigators have also proposed a lot of helpful numerical strategies for dealing with computational implementations of softening models [28,29].

The other topic to be studied is strain-rate dependency of progressive failures in quasi-brittle media, which strongly couples with the post-peak softening. For concrete, the peak strength increases and the fracture pattern becomes more scattering with increasing strain rate [30]. By proposing the extended Confinement Shear Lattice (CSL) model, Cusatis [31] investigated the effects of both the rate of crack opening and the viscoelastic deformation of the unfractured cement paste on concrete strength and fracture behavior. Wu et al. [32] made experimental observations and concluded that under intermediate strain rates, the concrete tensile strength depended on the type of tests, which cannot be captured by the Weibull effective volume method. The strain-rate sensitivity of concrete tensile failure was recurred by adopting the damaged plasticity theory combined with the strain-rate effect to describe the dynamic mechanical behavior of mortar matrix [33]. Xu and Wen [34] conducted theoretical and experimental studies on the nonlinear mechanical properties of lightweight foamed concrete under uniaxial compression over a temperature range of 223–343 K and a strain rate range of 0.001–118 per second.

In this study, both softening and strain-rate sensitivity are modeled by accounting for the viscous effect due to local breakage. It is conducted within the framework of lattice-type modeling, but the methodology can be easily extended to other computational models. We are taking the influence of ongoing local breakages on the remaining specimen as viscoelastic. When some element reaches its strength limit, its capability of bearing load vanishes suddenly and completely or significantly, leading to an immediate release of internal force in this cracking element. Such an immediate released internal force can be taken as a force suddenly applied on the remaining specimen. Therefore, the viscous effect can play an important role, i.e. the corresponding deformation of the remaining specimen somehow delays as compared with the release of internal forces of cracking elements. Based on such a physical picture, we construct a new algorithm based on a new concept, i.e. viscous force (VF) field, to deal with the above viscoelastic process. In the theoretical framework, characteristic times of both VF release and external loading are accounted for, leading to the capability of dealing with multi-time-scale problems [25].

A stepwise force-*time* constitutive law to be adopted here is different from the force-*deformation* ones (e.g., [17,28,35,36]). In literature, to keep algorithms to be sequentially linear, the original nonlinear force-*deformation* constitutive law is often approximated as saw-tooth shaped or piece-wise linear. The latter may be a closer approximation for an originally smooth curve. In both cases, the caused error can be overcome by adopting finer constitutive discretizations. An improved version was developed in [35]. In this study, the physical picture aforementioned, where VF evolves with time (instead of deformation), leads to a *temporal* constitutive law. This law is discretized stepwise, inspired by the saw-tooth concept [28].

This paper is presented as follows. In Section 2, the GB lattice model [13,37] is briefly introduced as the mesoscale model, including a failure criterion called the modified Mohr–Coulomb rule. In Section 3, we adopt a 1D bar system to illustrate the storage and release of viscous force fields, the release of internal forces of cracking elements and their coupling mechanisms. Particularly, two kinds of stress redistributions, i.e. one due to the release of internal forces in breaking elements and the other due to the release of viscous force fields (or equivalently due to the viscous delay in the deformation of the remaining specimen), are shown in details. Subsequently, the general procedure of the proposed method, i.e. the event-driven and non-