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Computing apparent planetary magnitudes for *The Astronomical Almanac*

A. Mallama [a,*], J.L. Hilton [b]

[a] 14012 Lancaster Lane, Bowie, MD, 20715, USA
[b] US Naval Observatory, 3450 Massachusetts Avenue, NW, Washington, DC, 20392, USA

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ABSTRACT

Improved equations for computing planetary magnitudes are reported. These formulas model V-band observations acquired from the time of the earliest filter photometry in the 1950s up to the present era. The new equations incorporate several terms that have not previously been used for generating physical ephemerides. These include the rotation and revolution angles of Mars, the sub-solar and sub-Earth latitudes of Uranus, and the secular time dependence of Neptune. Formulas for use in *The Astronomical Almanac* cover the planetary phase angles visible from Earth. Supplementary equations cover those phase angles beyond the geocentric limits. Geocentric magnitudes were computed over a span of at least 50 years and the results were statistically analyzed. The mean, variation and extreme magnitudes for each planet are reported. Other bands besides V on the Johnson–Cousins and Sloan photometric systems are briefly discussed. The planetary magnitude data products available from the U.S. Naval Observatory are also listed. An appendix describes source code and test data sets that are available online for computing planetary magnitudes according to the equations and circumstances given in this paper. The files are posted as supplementary material for this paper. They are also available at SourceForge under project https://sourceforge.net/projects/planetary-magnitudes/ under the 'Files' tab in the folder 'Ap_Mag_Current_Version'.

1. Introduction

Apparent magnitudes are an essential element of planetary physical ephemerides. They are generally computed along with other physical quantities such as the sub-solar latitude and the phase angle. An associated measure called 'surface brightness' (usually given in magnitudes per square arc-second) is critical for planning observations where an exposure time must be computed in advance. One example is remote observation from spacecraft where commands must be uploaded ahead of the planned observation. Brightness data are required any time that a signal-to-noise ratio is needed. Apparent magnitudes are also widely listed in almanacs such as *The Astronomical Almanac*, magazines intended for amateur astronomers, newspaper articles for the general public, as well as in astronomical observers' guides and astronomy textbooks. More recently, on-line ephemerides such as the U.S. Naval Observatory's Topocentric Configuration of Major Solar System Bodies (USNO, 0000), HORIZONS (Giorgini et al., 1996) and self-contained software such as the Multiyear Interactive Computer Almanac (MICA, 2005) have also been providing apparent planetary magnitudes. Finally, the direct detection of exo-planets depends on their apparent magnitudes, which may be estimated from their solar system counterparts.

In most cases the apparent magnitude refers to that on the 'visual system'. The *visual magnitude* is an old term referring to observations made with the human eye principally during the era before electronic sensors became available. Nowadays the visual magnitude is commonly taken to mean the V-band of the Johnson–Cousins photometric system (Johnson and Morgan, 1953; Cousins, 1976a, b). The response curve of that band is centered at 0.549 µm and has a full-width-at-half-maximum of 0.086 µm. Thus, it is somewhat like the response curve of the human eye. Magnitudes herein are taken to be on the V-band unless otherwise indicated.

The purpose of this paper is to specify formulas for computing apparent magnitudes of solar system planets based upon the latest models and the most complete sets of observations available. Section 2 briefly reviews the literature on planetary magnitudes and then describes how apparent magnitudes are computed. Section 3 discusses the apparent magnitude of each planet individually and presents the equations for computing their apparent brightness. Section 4 lists statistics of the apparent magnitudes, such as mean opposition values, brightest and faintest and the

* Corresponding author.
E-mail addresses: anthony.mallama@gmail.com (A. Mallama), james.l.hilton1@navy.mil (J.L. Hilton).

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greatest brilliancy of Venus. Section 5 provides an overview of other wavelength bands besides V in the Johnson–Cousins system that may be useful to observers. The Sloan photometric system is also described because it is becoming the new standard. Section 6 lists the planetary magnitude data products that are available from the U.S. Naval Observatory. Section 7 summarizes the paper and presents our conclusions. An appendix describes source code and test data sets for computing planetary magnitudes. The files are hosted on-line at SourceForge which is an open-source software site.

2. Planetary magnitudes

Müller (1893) developed Eq. (1), a general-purpose formula for predicting apparent magnitudes of the planets. The apparent magnitude depends upon the planet's distance from the Sun, r, and from the Earth, d, in accordance with the inverse square law. Another important factor is the illumination phase angle, α, which is defined as the arc between the Sun and the sensor with its vertex at the planetocenter. Thus, small values of α correspond with more fully illuminated disks and large values of α to thin crescents.

$$V = 5\log_{10}(rd) + V_1(0) + C_1\alpha + C_2\alpha^2 + \cdots \quad (1)$$

V is the apparent visual magnitude, and $V_1(0)$ is the magnitude when observed at $\alpha = 0$ and when the planet is at a distance of one au from both the Sun and the observer. $V_1(0)$ is sometimes referred to as the planet's *absolute magnitude* or *geometric magnitude* and it may also be thought of as $C_0\alpha^0$. The sum $\Sigma_n C_n\alpha^n$ is called the phase function. The phase function generally increases the planet's apparent magnitude with increasing phase angle.

Nearly the entire 180° of the phase curves for Mercury and Venus have been observed. Fig. 1 shows that the brightness of airless Mercury declines dramatically with phase angle while that of cloud-covered Venus drops off less sharply. Mars and the Earth are intermediate cases between the extremes of Mercury and Venus. The Earth-viewable ranges of α for the giant planets are restricted, from about 12° for Jupiter to less than 2° for Neptune. The phase functions of Jupiter and Saturn have been determined accurately over their entire observable ranges. The magnitude changes for Uranus and Neptune as seen from the Earth arising from phase angle is less than 0.01 magnitude. So, the phase functions can be ignored for the purposes of computing apparent geocentric ephemeris magnitudes for these two planets. The phase functions of the giant planets for large values of α, shown in Fig. 2, are based upon measurements obtained from interplanetary spacecraft.

More than 50 years ago Harris (1961) summarized the available observations and analyses of planetary brightness. Those studies began in the 1800s with visual magnitudes actually estimated by the human eye. Filtered photoelectric photometry only became standardized when Johnson and Morgan (1953) specified the characteristics of the V-band and established a set of reference stars with accurately calibrated magnitudes. Only the earliest V-band magnitudes of the planets were available to Harris.

The equations given by Harris are now outdated. They do not capture all the significant aspects of planetary brightness and variability observed in the six decades since his work was published. Subsequent observations include a wider range of physical and geometrical conditions as well as the longer time span of observation. Many subtleties also remained hidden until more accurate observations were obtained from space-based instruments and from ground-based telescopes using CCD sensors.

Mallama et al. (2017) reported up-to-date values of $V_1(0)$ for all the planets and listed the best fitting coefficients of α. They also evaluated more observational circumstances and listed coefficients for other parameters such as rotation angle, inclination and time period, where needed. The following synopsis mentions a few of the more striking geophysical implications especially as they pertain to apparent magnitudes.

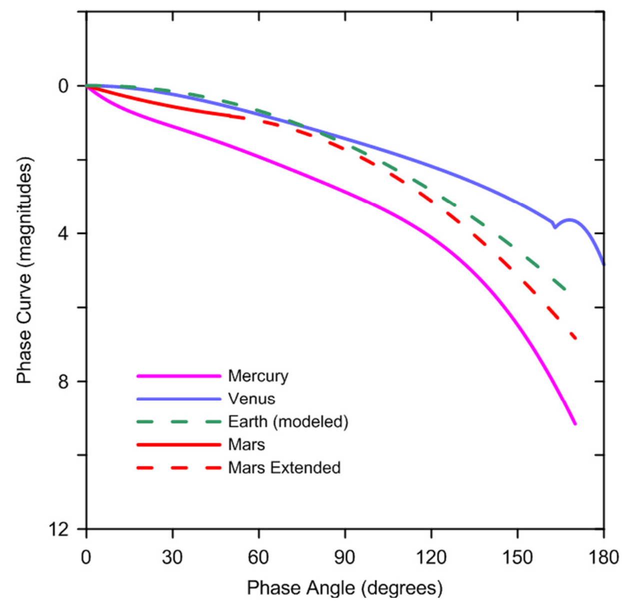

Fig. 1. The phase curve for Mars with its thin atmosphere lies between those of barren Mercury and cloud-shrouded Venus. The upturns in the phase functions of Mercury and Mars near $\alpha = 0°$ are due to strong backscattering from their surfaces. Forward scattering by liquid droplets in Venus' atmosphere is the source of the inflection point in the phase curve of Venus at about 163°. The dashed lines for the Earth and Mars are discussed in Sections 3.3 and 3.4, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

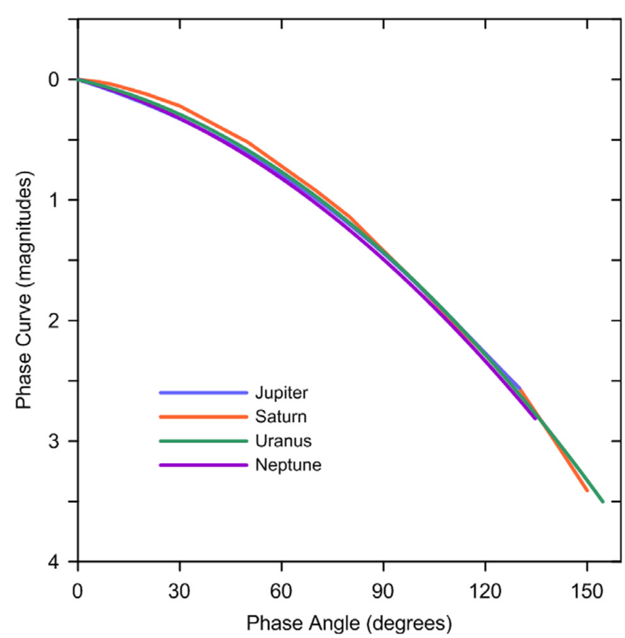

Fig. 2. The phase curves for the giant planets are more similar to one another than are those of the terrestrial bodies. The large phase angle data are based upon observations obtained from interplanetary spacecraft as explained in Sections 3.5 through 3.8.

Mercury was observed with the SOlar and Heliospheric Observatory (SOHO) spacecraft, which allowed coverage of a greater range of phase angles than ever before, extending over $2° < \alpha < 170°$. Mallama et al. (2002) analyzed those magnitudes along with ground-based CCD data. Mercury's large brightness surge