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Effectivity and efficiency of selective frequency damping for the computation of unstable steady-state solutions

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ABSTRACT

Selective Frequency Damping (SFD) is a popular method for the computation of globally unstable steady-state solutions in fluid dynamics. The approach has two model parameters whose selection is generally unclear. In this article, a detailed analysis of the influence of these parameters is presented, answering several open questions with regard to the effectiveness, optimum efficiency and limitations of the method. In particular, we show that SFD is always capable of stabilising a globally unstable systems ruled by one unsteady unstable eigenmode and derive analytical formulas for optimum parameter values. We show that the numerical feasibility of the approach depends on the complex phase angle of the most unstable eigenvalue. A numerical technique for characterising the pertinent eigenmodes is presented. In combination with analytical expressions, this technique allows finding optimal parameters that minimise the spectral radius of a simulation, without having to perform an independent stability analysis. An extension to multiple unstable eigenmodes is derived. As computational example, a two-dimensional cylinder flow case is optimally stabilised using this method. We provide a physical interpretation of the stabilisation mechanism based on, but not limited to, this Navier–Stokes example.

1. Introduction

Understanding fluid flow instabilities is of fundamental importance for laminar-turbulent transition and for flow control. Flow instabilities can be characterised through linear stability analysis, for which it is necessary to obtain an accurate representation of the laminar base flow solution [1,2]. Letting f be the non-linear Navier–Stokes operator applied to a state variables $\boldsymbol{q}$ vector, with adequate boundary and initial conditions, the Navier–Stokes equations can be written as

$$\dot{\boldsymbol{q}} = f(\boldsymbol{q}), \tag{1}$$

where the dot expresses the time derivative. The steady state of eq. (1) satisfies $\dot{\boldsymbol{q}}_{\boldsymbol{s}} = f(\boldsymbol{q}_{\boldsymbol{s}}) = 0$. The stability approach relies on decomposing the instantaneous flow field into the base flow $\boldsymbol{q}_{\boldsymbol{s}}$ plus a time-dependent infinitesimally small perturbation field $\boldsymbol{q}'$ such that, in a two-dimensional space,

$$\boldsymbol{q}(\boldsymbol{x}, t) = \boldsymbol{q}_s(\boldsymbol{x}) + \epsilon_{\mathrm{A}}\boldsymbol{q}'(\boldsymbol{x}, t), \quad 0 < \epsilon_{\mathrm{A}} \ll 1, \tag{2}$$

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where $\boldsymbol{x}$ represents the spatial coordinates and t the time [2,3]. Stability analysis requires an accurate solution of the base flow as the stability properties critically depend on its spatial derivatives.

The difficulty in computing time-independent Navier–Stokes solutions arises for globally unstable flow fields, inasmuch as the instantaneous flow naturally diverges from the steady state [4]. To overcome this obstacle, mainly two numerical methods are employed in the literature: Newton iteration methods [5] are the classical approach; however, these methods may have severe practical limitations due to the sensitivity to the initial guess and the required computational cost for large and strongly nonlinear systems [6,7]. In the past years, the Selective Frequency Damping (SFD) method developed by Åkervik et al. [4] has arisen as a solid alternative. Its robustness and ease of implementation into existing time-stepping methods have made it increasingly popular to the point that it is generally the preferred method for aeronautical applications [8].

Based on control theory, SFD adds a linear forcing term to eq. (1), which drives the flow field $\boldsymbol{q}$ towards $\boldsymbol{q}_s$ [4]. As this target solution is not known beforehand, $\boldsymbol{q}_s$ is substituted by a low-pass filtered version of $\boldsymbol{q}$, denoted by $\bar{\boldsymbol{q}}$. The evolution equation for $\boldsymbol{q}$ is then rewritten as

$$\dot{\boldsymbol{q}} = f(\boldsymbol{q}) - \chi(\boldsymbol{q} - \bar{\boldsymbol{q}}), \qquad \chi \in \mathbb{R}^+. \tag{3}$$

The forcing is a linear reaction term proportional to the high-frequency content of the flow. Its effectiveness in quenching unstable frequencies and hence suppressing the associated instabilities depends on the feedback control coefficient χ. Åkervik et al. [4] suggest an exponential kernel filter to compute $\bar{\boldsymbol{q}}$. Since the implementation of the integral formulation of the filter would generally imply infeasible memory requirements in practice, its differential formulation is considered instead:

$$\dot{\bar{\boldsymbol{q}}} = \frac{\boldsymbol{q} - \bar{\boldsymbol{q}}}{\Delta}, \qquad \Delta \in \mathbb{R}^+. \tag{4}$$

The time constant Δ is related to the cut-off frequency (ω_c) of the low-pass filter through $\Delta = 1/\omega_c$. The performance of SFD depends on the two aforementioned parameters of the model, χ and Δ, which must be chosen as inputs of the simulation. Appropriate values depend on the flow problem and hence their selection is key to the method's effectiveness and efficiency as they determine the stability and convergence rate [4,6,9,10]. Not every combination of χ and Δ guarantees that the flow field is driven towards the steady state, and even if so, the required computational time may be so large that the approach is impractical. Hence, how to select adequate χ and Δ is a common predicament in the literature.

SFD has nonetheless been very successfully applied to two- and three-dimensional flow configurations: Åkervik et al. [4] first applied the method for stabilising a separation bubble with success and the steady solution of a confined separated flow was obtained by Åkervik et al. [11]. Pier [12] computed the base flow around a sphere to analyse local and global instabilities developing in the wake. Schmid [13] analysed the stability of the flow in a square cavity by using a reference solution computed with SFD. Bagheri et al. [14] successfully applied SFD to stabilise a jet in crossflow and Fani et al. [15] found the base flow for a three-dimensional T-mixer. Loiseau et al. [16] studied roughness-induced transition by performing stability analyses using base-flow solutions computed with SFD. More recently, Richez et al. [8] applied SFD to stabilise RANS simulations of the turbulent separated flow around an airfoil at stall and Kurz and Kloker [17] computed with SFD the steady state of a three-dimensional boundary layer over a swept wing with roughness elements. Significant contributions to the advancement of the methodology where published by Jordi et al. [9], who developed an alternative SFD formulation, and Cunha et al. [6], who developed an optimisation method for SFD simulations based on Dynamic Mode Decomposition (DMD).

The model parameters χ and Δ are commonly based on rough estimations introduced by Åkervik et al. [4] or on parametric studies of simplified models. Jordi et al. [9,10] used a scalar model problem to infer the behaviour of SFD when applied to a real flow problem. By using this model, Jordi et al. [9] generate stability curves identifying the influence of χ and Δ and indicate that SFD is incapable of stabilising steady unstable eigenmodes, corroborating the results of Vyazmina [18]. It was observed that increasing χ may not always guarantee convergence, contrary to the consensus introduced by Åkervik et al. [4]. Jordi et al. [10] hypothesised that the parameter values that optimise the scalar model problem also optimise the full flow problem and developed a coupled approach, which combines the computation of partially converged flow fields, stability analyses and parameter optimisation for the model problem. Cunha et al. [6] use DMD of the controlled flow field to determine parameters that minimise the growth rate of the least stable DMD mode.

There are cases in which SFD reportedly failed [18,19]. It was claimed that cases in which the flow field presents steady unstable eigenmodes, SFD is unable to drive the simulation towards the steady state. Several authors indicate that too large χ yield infeasible simulation times [4,6,19]. Massa [20] and Teixeira and Alves [7] reported that stabilising a flow field that is unstable to more than one eigenmode can be a challenging task. In particular, Massa [20] claimed that SFD fails to converge towards the base flow if unstable eigenvalues with high amplification rates and low-frequency weakly unstable eigenvalues are both present in the flow field. In conclusion, a better understanding of χ and Δ is required to establish the feasibility of the method in the first place.

In this paper, the role played by χ and Δ is analysed in detail. We aim to answer the main open question presented by literature:

Which values of χ and Δ are effective and most efficient?