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Technical Note

# Developing laminar natural convection of power law fluids in vertical open ended channel

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ABSTRACT

Steady, developing laminar natural convection heat transfer for power law fluids between two parallel vertical plates where the fluid is entrained from the bottom and exiting from top is studied. The results for asymmetric and symmetric configurations of the heated plates, which are kept at uniform temperatures with an aspect ratio of 40, are presented. For the numerical computations, the ANSYS Workbench Fluent commercial package is used to solve the governing equations and its numerical output is post-processed. Various range values of Rayleigh number ($Ra = 10^4, 10^5, 10^6$) Prandtl number ($Pr = 10, 100, 1000$), non-Newtonian power law indexes ($0.6 \leqslant n \leqslant 1.4$) of the fluids and the temperature ratios of colder and hotter walls ($r_T = 0, 0.5, 1.0$) are considered. The temperature and velocity profiles, and the average Nusselt number distributions are presented. Comparisons between Newtonian and non-Newtonian fluids are made in terms of the variations of the average Nusselt number as a function of the non-dimensional governing parameters.



## 1. Introduction

There has been much research on natural convection heat transfer explaining its characteristics and presenting the heat transfer correlations related to a given set of parameters. Among those studies, some researchers investigated natural convection of Newtonian fluids within parallel plates [1–9], where the fluid enter from the bottom of channel and elevate by buoyancy force. Aung [2] studied the developing laminar free convection between vertical flat plates with asymmetric heating. Aung [2] considered a non-uniform inlet velocity profile at the entrance. M. Thebault, S. Giroux-Julien and other investigators [5–9] concentrated on natural convection within two parallel plates for Building Integrated Photovoltaic (BIPV) systems application by using 3-D model for transitional and turbulent numerical analysis of Newtonian fluids in order to study turbulent natural convection within a narrow channel. Newtonian and non-Newtonian fluid application can be found in oil-drilling, pulp paper, and polymer processing. Recent studies for non-Newtonian fluids that obey the power law model between parallel plates [10,11], rectangular [12], square [13] or square eccentric duct annuli [14] enclosures are investigated. However, there were few open-ended channels studies other than closed cavity scenarios. In this present work, a steady, developing, laminar natural convection heat transfer for power law fluids within an open-ended channel between two parallel vertical plates where the fluid is entrained from the bottom and exiting from top, is studied.

In summary, in studies related to non-Newtonian power law fluids, more closed systems as opposed to open systems were investigated along with low aspect ratio cases. Therefore, in this work, in order to obtain further insights on laminar natural heat transfer not only for Newtonian fluid but also for non-Newtonian power law fluids, a thorough numerical analysis is carried out using ANSYS Workbench Fluent for different scenarios of Rayleigh number $Ra$, Prantdl number $Pr$, and power law index $n$ as well as heating configurations with larger aspect ratios. The main goal of this paper is to show natural convection characteristics for both Newtonian and non-Newtonian fluid in a high aspect ratio open system and compare results and trends to those of Newtonian case with the same configuration in different Rayleigh number, Prandtl number and heating configurations.

## 2. Governing equations

The flow channel consists of two vertical plates with heights, $H$, and channel width, $D$. The inlet and outlet of the channel are at the top and bottom respectively, allowing fluid to enter from the bottom with temperature $T_0$ and exiting from the top after being

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**Nomenclature**

| | |
|---|---|
| UWT | uniform wall temperature |
| UHF | Uniform Heat Flux |
| $x$ | X-direction |
| $y$ | Y-direction |
| $H$ | channel height |
| $D$ | channel width |
| $u$ | velocity components at X-direction |
| $v$ | Velocity components at Y-direction |
| $u_0$ | arbitrary reference velocity |
| $T$ | temperature of fluid |
| $T_0$ | temperature of environment |
| $p$ | fluid pressure |
| $\rho$ | density of fluid |
| $t$ | time |
| $g$ | gravitational acceleration |
| $c_p$ | specific heat at constant pressure |
| $\mu$ | dynamic viscosity of fluid |
| $\alpha$ | thermal diffusivity of fluid |
| $\beta$ | thermal expansion coefficient |
| $v$ | kinematic viscosity of fluid |
| $k$ | thermal conductivity of fluid |
| $K$ | consistency index |
| $n$ | power law index |
| $Pr$ | Prandtl number |
| $Gr$ | Grashof number |
| $Ra$ | Rayleigh number |
| $Re$ | Reynolds number |

*Subscripts*

| | |
|---|---|
| $\infty$ | free stream |
| 1, 2 | hot and cold wall |

heated. The vertical walls are kept at uniform wall temperatures (UWT) $T_1$ and $T_2$.

The continuity, momentum and energy equations describing the unsteady, laminar, two-dimensional flow field and temperature distributions in the rectangular coordinate system, where the y-axis is along the vertical plate and opposite to the gravitational direction, are as follows:

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0 \quad (1)$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\rho g_x-\frac{\partial p}{\partial x}+\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}\right) \quad (2)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\rho g_y-\frac{\partial p}{\partial y}-\left(\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}\right) \quad (3)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{1}{\rho c_p}\phi \quad (4)$$

where $u$, $v$ represents the velocity components in the $x$, $y$ directions, respectively, $T$ is the temperature, $\rho$ is the density of the fluid, $\mu$ is the dynamic viscosity, $\beta$ is the thermal expansion coefficient, $v$ is the kinetic viscosity of the fluid and $\phi=2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]$, which is the viscous heating; $\alpha=\frac{k}{\rho c_p}$, which is the thermal diffusivity of the fluid.

For a non-Newtonian fluid following the Ostwald–de Waele power law model [14], shear stress can be represented in tensor form as follows,

$$\tau_{xy}=2\mu_a D_{xy}=\mu_a\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \quad (5)$$

where $D_{ij}$ is the rate of deformation tensor for the two-dimensional Cartesian coordinates, $\mu_a$ is the apparent viscosity defined as,

$$\mu_a=K\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right\}^{\frac{n-1}{2}}. \quad (6)$$

Here, $K$ is the consistency index and $n$ is the power law index. The shear stress and apparent viscosity are reduced following an order of magnitude analysis as follows,

$$\tau_{xy}=\mu_a\frac{\partial v}{\partial x} \quad (7)$$

$$\mu_a=K\left(\frac{\partial v}{\partial x}\right)^{n-1} \quad (8)$$

When $n=1$, the fluid becomes a Newtonian fluid; when $n<1$, the fluid is defined as shear thinning fluid (or pseudoplastic fluid); when $n>1$, the fluid is defined as shear thickening fluid (or dilatant fluid).

The no slip boundary conditions at two the sidewalls are,

$$u, v=0 \text{ at } x=-\frac{D}{2},\frac{D}{2} \quad (9)$$

Corresponding thermal boundary conditions at the two sidewalls are,

$$T=T_1 \text{ at } x=-\frac{D}{2} \quad (10)$$

$$T=T_2 \text{ at } x=\frac{D}{2} \quad (11)$$

For a steady-state flow with negligible thermal dissipation, the following dimensionless variables are defined as,

$$x^*=\frac{x}{D}, y^*=\frac{y}{D}, u^*=\frac{uD}{\alpha}, v^*=\frac{vD}{\alpha}, \Theta^*=\frac{T-T_\infty}{T_2-T_\infty} \quad (12)$$

Thus, the governing equations and their boundary conditions in terms of the non-dimensional parameters become,

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \quad (13)$$

$$u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=RaPr\Theta^*+Pr\left(\frac{\partial}{\partial x^*}\left(\frac{\mu_a}{K}\frac{\partial v^*}{\partial x^*}\right)\right) \quad (14)$$

$$u^*\frac{\partial \Theta^*}{\partial x^*}+v^*\frac{\partial \Theta^*}{\partial y^*}=\frac{\partial^2\Theta^*}{\partial x^{*2}} \quad (15)$$

$$u^*, v^*=0 \text{ at } x^*=-\frac{1}{2},\frac{1}{2} \quad (16)$$

$$\Theta^*=\Theta_1^* \; at \; x^*=-\frac{1}{2} \quad (17)$$

$$\Theta^*=\Theta_2^* \; at \; x^*=\frac{1}{2} \quad (18)$$

where $\Theta^*$ is defined as $\Theta^*=\frac{T-T_\infty}{T_2-T_\infty}$, which represents the non-dimensional temperature. For the verification of numerical model,