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Noise influence on dissipative solitons in a chain of active particles

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HIGHLIGHTS

- The total number of solitons in the chain depends on the noise strength.
- Modes probability distributions are formed by two mechanisms.
- The first mechanism determines the living time of the modes.
- The second mechanism governs switching between modes.
- Analytical approach and numerical simulations is obtained.

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ABSTRACT

The dynamics of a chain of interacting active particles of Rayleigh-type is studied in presence of noise. Particles are interconnected via Morse potential forces. In the noiseless scenario, the steady-state modes (attractors) of the chain with periodic boundary conditions look like cnoidal waves with uniform distribution of the particles density maxima along the chain. The distribution of modes probabilities shifts from one with prevailing cnoidal waves to one with prevailing optical mode under noise influence. We propose an analytical approach modelling the noise effect in good agreement with the numerical simulation.

1. Introduction

Active matter systems consist of elements (active particles, self-propelled particles, microswimmers and others) which are able to take energy from their environment and transform it to energy of ordered motion driving themselves far from equilibrium [1–9]. Because of this property they support a series of new behaviours which are not attainable by matter at thermal equilibrium, such as swarming formation and another collective effects [10]. Objects with possibility to self-propulsion are important examples of active matter. Firstly self-propelled particles were introduced by Reynolds [11] to simulate swarm behaviour of animals at the macroscale, partially the aggregate motion of flocks of birds, herds of land animals, and schools of fish. Also a lot of models exists which simulate behaviour of swimmers (microswimmers) [12,13], active colloids [14,15] and so on.

Self-propelled Brownian particles, in particular, have come under the spotlight of the physical and biophysical research communities [16]. These active particles are biological or artificial microscopic and nanoscopic objects which can propel

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themselves by taking up energy from their environment and converting it into kinetic energy [17]. The motion of passive Brownian particles is driven by equilibrium thermal fluctuations due to random collisions with the surrounding fluid molecules [18]. But self-propelled Brownian particles exhibit an interplay between random fluctuations and active swimming that drives them into a far-from-equilibrium state [10,19,20].

Recently has been demonstrated that many aspects of complex organization effects can be studied in physical models based on a minimal set of rules or interactions, leading to the emergence of the new kind of active matter [21]. The last is defined as a non-equilibrium matter in which energy uptake, dissipation and movement take place at the level of discrete microscopic constituents.

For example, a dense swarm of bacteria can behave collectively as "living" fluid, showing self-organize in complex regular patterns, turbulent motion, or "freeze" into a solid-like bio-film [22–26]. This type of behaviour is well known in usual non-animate matter that exhibits phase transitions under influence of external temperature or some external forces (for example, system perturbation at its boundaries, an electric or magnetic field and so on).

The studying of dynamics of active Brownian particles ensemble including potential interactions [1–4,27,28] is the main point of paper presented. The principal interest is focused on the influence of heating (i.e. stochastic external force) on ensemble dynamics. A number of works in this area was carried out with using of the popular Toda–Rayleigh model. In this model point particles with Rayleigh-type negative friction are connected to each other by Toda potential which are similar Morse potential forces at not very high level of energy. [6–8,29–31].

Steady-state modes (attractors) of a chain with periodic boundary conditions look like cnoidal waves with a uniform spatial distribution of maxima velocity of particles which may be considered as an ordered ensemble of dissipative (discrete) solitons [32] or autosolitons. Different modes have different number of solitons and average velocity.

This work is organized as follows. In Section 2 we present potential function, nonlinear friction and equations describing the system while in Section 3 we describe method of modes determinations. In Section 4 we consider transformation of stationary modes distribution due to noise changing. In Section 5 we examine how noise changes lifetimes of modes with different number of solitons. Section 6 is devoted to statistics of noise induced jumps between modes.

2. Model and equations of dynamics

We consider an one-dimensional lattice of point masses with nonlinear active friction interacting via potential Morse forces. The dynamics of an isolated particle is described by equation

$$m\ddot{x} - \gamma_0(1 - \frac{\dot{x}^2}{v_0^2})\dot{x} = 0, \tag{1}$$

where m is the mass of the particle, γ_0 is the negative friction coefficient and v_0 is the particle steady-state velocity. Particles form a chain if each of them is connected to two neighbours via potential forces possessing a minimum of potential. In particular, we consider the Morse potential

$$U(r) = D(e^{-2b(r-\sigma)} - 2e^{-b(r-\sigma)}). \tag{2}$$

Here and below D is the Morse potential well depth, that is $U = -D$ at $r = \sigma$, b is the potential stiffness. Then dynamics of a chain is described by the following equations of motion in dimensionless variables

$$\begin{aligned} &\ddot{q}_n - \mu(1 - \frac{\dot{q}_n^2}{v^2})\dot{q}_n \\ &= (1 - e^{q_n - q_{n+1}})e^{q_n - q_{n+1}} - (1 - e^{q_{n-1} - q_n})e^{q_{n-1} - q_n} \\ &+ \sqrt{2D_E}\xi_n(t), \end{aligned} \tag{3}$$

where $q_n = b(x_n - n\sigma)$ is the dimensionless deviation of nth particle from its equilibrium state given by $x_{n0} = n\sigma$, σ denotes the equilibrium distance between particles. The frequency of small oscillations of a particle near the minimum of the Morse potential is given by $\omega_M = \sqrt{\frac{2Db^2}{m}}$, hence the dimensionless time is taken as $\tau = \omega_M t$ whereas the dimensionless velocity is given by $v = v_0 b/\omega_M$. D_E is noise intensity and $\xi_n(t)$ describes a source of the Gaussian white noise [6,33]. Finally, we define $\mu = \gamma_0/m\omega_M$. Dynamics of the limited length chain with periodic boundary conditions is studied.

It is assumed $v_0 = 1$ in simulations. In this work we use values of parameters: $\mu = 1$, $b = 3$, total number of particles $N = 20$.

The equations are solved numerically by the Runge–Kutta fourth order method with control of the calculations accuracy. Simulation timestep $\Delta t = 5 \cdot 10^{-3}$.

3. Switching of modes due to noise

The total number of modes in a chain composed by N particles is $N + 1$ as following: two non oscillatory modes corresponding to motion of the chain as a whole in both directions, and $N - 1$ oscillatory modes. If N is even, a so-called optical mode with opposite signs of velocity of adjacent particles is included. Each of $N + 1$ modes can be excited selectively by appropriate choice of initial conditions [6,7,30].