Local detrended cross-correlation analysis for non-stationary time series

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HIGHLIGHTS

- LDCCA method is proposed to quantify temporal characteristics of coupled time series.
- The performance of LDCCA is validated with typical non-stationary time series.
- The LDCCA is used to uncover the local evolution dynamics of gas–liquid churn flows.

ARTICLE INFO

ABSTRACT

We propose a method called local detrended cross-correlation analysis (LDCCA) to quantify temporal power-law cross-correlation characteristics of coupled time series at local samples. The proposed method is validated with uncoupled Gaussian white noises, coupled ARFIMA processes and Hénon maps. As an example, electrical probe technologies are employed to detect the flow structure information of gas–liquid churn flows in a vertical pipe, and temporal cross-correlation characteristics of flow interfacial structures are investigated using the proposed LDCCA. The results show that the proposed LDCCA can provide beneficial insights to local dynamic evolution behaviors of the flow interfacial structures.

1. Introduction

Since complex systems always consist of interacting constituents with a coupling relationship, the cross-correlation analysis of these constituents is beneficial to diagnose and understand the whole system. Analysis of a power-law cross-correlation between various time series has become a popular topic [1–4]. Podobnik and Stanley [5] first proposed a method of detrended cross-correlation analysis (DCCA) to investigate the power-law cross-correlation between different simultaneously recorded time series. The DCCA shows a good performance in quantifying a power-law cross-correlation of different time series even in the presence of non-stationary periodic trends [6,7]. Kristoufek [8–10] theoretically analyzed the relationship between bivariate Hurst exponent and separate Hurst exponents, and attempted to provide interpretation and practical implication of the bivariate Hurst exponent.

Importantly, Zebende et al. [11,12] proposed a DCCA cross-correlation coefficient to quantify cross-correlation level of coupled time series. The DCCA has been widely used in various fields, as in climatology [13,14], finance [15–19], neuro-science [20], traffic [21], turbulent flow [22] and others. Meanwhile, selected scholars paid their attention to multifractal detrended cross-correlation analysis [23–25]. The development of DCCA has motivated others in introducing alternative methods such as the detrending moving-average cross-correlation analysis (DMCA) [26–28] and the height cross-correlation

analysis (HXA) [29]. Based on the DCCA, the DMCA and the HXA, Kristoufek [30] proposed three power-law coherency parameters for studying power-law cross-correlations between simultaneously recorded time series. In addition, a method of detrended partial-cross-correlation analysis (DPCCA) was proposed [31] to quantify the intrinsic relations of two non-stationary signals with influences of other signals removed. Piao et al. [32] applied the DPCCA to analyze the temporal evolutions of intrinsic correlations between winter-time Pacific-Northern America pattern (PNA)/East Pacific wave-train (EPW) and winter-time drought in the west United States. Qian et al. [33] developed a multifractal DPCCA method and found it beneficial to reveal the hidden multifractal nature of time series.

Although cross-correlation analysis of interactive signals from complex systems has achieved a great development in the past decade, within our knowledge there are seldom methods to investigate the temporal cross-correlation characteristics of coupled time series. In this current study, as a generalization of the DCCA [5], a method of local detrended cross-correlation analysis (LDCCA) is proposed to investigate the temporal cross-correlation behavior of time series at local samples. The performance of the proposed LDCCA is evaluated using uncoupled Gaussian white noises, coupled ARFIMA processes and Hénon maps. As an example, the LDCCA is employed to investigate the temporal evolution characteristics of the interfacial structures in gas–liquid churn flows.

This paper is structured as follows: Section 2 describes the algorithm of the LDCCA; Section 3 presents the evaluation of the LDCCA performance; Section 4 presents the temporal cross-correlations of changeable interfacial structures in churn flows using LDCCA.

2. Algorithm of DCCA and LDCCA

The algorithm of DCCA [5] can be briefly introduced as follows. For given time series $\{x_i\}$ and $\{y_i\}$ with lengths of N, we first compute two integrated signals as follows

$$R_k = \sum_{i=1}^{k} x_i, R'_k = \sum_{i=1}^{k} y_i, k = 1, 2, \ldots, N \tag{1}$$

Then, the series R_k and R'_k are divided into $N - n$ overlapping boxes, and each box contains $n + 1$ values. For a box from i to $i + n$, we define the local trend, $\widetilde{R}_{k,i}$ and $\widetilde{R}'_{k,i}$, to be the ordinate of a linear least-squares fit. Next the covariance of the residuals in each box is calculated using the following equation

$$f_{\text{DCCA}}^2(n, i) \equiv \frac{1}{n+1} \sum_{k=i}^{i+n} (R_k - \widetilde{R}_{k,i})(R'_k - \widetilde{R}'_{k,j}) \tag{2}$$

Finally, the detrended covariance function is calculated by summing over all overlapping $N - n$ boxes:

$$F_{\text{DCCA}}^2(n) \equiv (N - n)^{-1} \sum_{i=1}^{N-n} f_{\text{DCCA}}^2(n, i) \tag{3}$$

The power-law cross correlations between the time series $\{x_i\}$ and $\{y_i\}$ may exist only if $F_{\text{DCCA}}^2(n) \sim n^{2\lambda}$ for both series. The λ exponent quantifies the long-range power-law cross-correlation but does not quantify the level of cross-correlations. Zebende [11] proposed a DCCA cross-correlation coefficient ρ_{DCCA} to quantify the level of cross-correlation of the time series:

$$\rho_{\text{DCCA}} \equiv \frac{F_{\text{DCCA}}^2}{F_{\text{DFA}}\{x_i\}F_{\text{DFA}}\{y_i\}} \tag{4}$$

where $F_{\text{DFA}}\{x_i\}$ and $F_{\text{DFA}}\{y_i\}$ are the detrended root-mean-square functions of series $\{x_i\}$ and $\{y_i\}$, respectively. The cross-correlation coefficient ρ_{DCCA} is a dimensionless coefficient that ranges between -1 and 1. A value of $\rho_{\text{DCCA}} = 0$ means there is no cross-correlation between two the time series, whilst $0 < \rho_{\text{DCCA}} \leq 1$ and $-1 \leq \rho_{\text{DCCA}} < 0$ indicate positive and negative cross-correlation characteristics of both series respectively.

Based on the DCCA we propose a method of local detrended cross-correlation analysis (LDCCA) to investigate the temporal cross-correlation characteristics of coupled time series. The main idea of the LDCCA can be described as follows. A moving window ψ_{Ws} with a size of Ws is used to run along the entire coupled series. Having ψ_{Ws} centered at the sth samples of the coupled series, the DCCA is computed for all samples within the window ψ_{Ws}. The obtained cross-correlation coefficient is treated as the local detrended cross-correlation coefficient $\rho_{\text{LDCCA}}(s, Ws)$ due to the time scale Ws on sth samples. Through sliding the window one (or several) sample(s) each time and repeating the calculations until the end of coupled series, we can achieve the local cross-correlation detrended coefficient for time scale Ws. Next, Ws is increased and the aforementioned process is repeated again until a selected maximum Ws is attained. Details of LDCCA are described in Fig. 1.

(1) In Fig. 1(a), $X(s)$ and $Y(s)$ respectively indicate the sth samples of given time series $X(t)$ and $Y(t)$ with a same length L. For LDCCA, we first select a range $[Ws_{\min}, \ldots, Ws_{\max}]$ of the moving window sizes, i.e., the time scales.

For a window size Ws, padded series $X_{pad}(t)$ and $Y_{pad}(t)$ are obtained by adding $l = Ws/2$ reflected samples at the beginning and ending points of time series $X(t)$ and $Y(t)$. Here, the method of reflected-sample-padding is based on