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Eigentime identity of the weighted scale-free triangulation networks for weight-dependent walk

Meifeng Dai [a,*], Jingyi Liu [a], Jianwei Chang [b], Donglei Tang [c,*], Tingting Ju [a], Yu Sun [a], Weiyi Su [d]

[a] Institute of Applied System Analysis, Jiangsu University, Zhenjiang, 212013, PR China
[b] School of Finance and Economics, Jiangsu University, Zhenjiang, 212013, PR China
[c] School of Mathematics and Statistics, Nanjing Audit University, Nanjing, 211815, PR China
[d] Department of Mathematics, Nanjing University, Nanjing, 210093, PR China

HIGHLIGHTS

- Eigentime identity of the weighted scale-free triangulation networks.
- Transition weight matrix for weight-dependent walk.
- Recursive relationship of those eigenvalues of transition weight matrix.
- Eigentime identity grows sublinearly with the network order.

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ABSTRACT

The eigenvalues of the normalized Laplacian matrix of a network provide information on its structural properties and some relevant dynamical aspects, in particular for weight-dependent walk. In order to get the eigentime identity for weight-dependent walk, we need to obtain the eigenvalues and their multiplicities of the Laplacian matrix. Firstly, the model of the weighted scale-free triangulation networks is constructed. Then, the eigenvalues and their multiplicities of transition weight matrix are presented, after the recursive relationship of those eigenvalues at two successive generations are given. Consequently, the Laplacian spectrum is obtained. Finally, the analytical expression of the eigentime identity, indicating that the eigentime identity grows sublinearly with the network order, is deduced.

1. Introduction

In the recent years, the study of networks associated with complex systems has received much attention of researchers from different scientific fields, especially the weighted networks. As a standard tool, random walk on a network describes various dynamical processes in the network. The eigentime identity of weighted networks has gained much interest [1–5].

In the context of chemical physics, diverse problems are closely related to the eigenvalues and/or eigenvectors of the walk matrix. For example, the random target access time [6] is in fact the average trapping time for a special case of trapping problem [7], which is a fundament mechanism for various dynamical processes [8] and has attracted much attention from the physics community [9–13]. Since random walk is completely described by the transition matrix, most interesting

* Corresponding authors.
E-mail addresses: daimf@mail.ujs.edu.cn (M. Dai), tdonglei@nau.edu.cn (D. Tang).

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quantities and properties related to random walk is determined by the eigenvalues and eigenvectors of the transition matrix. Furthermore, the sum of reciprocals of each nonzero eigenvalues determines the random target access time.

In the past few years, there has been an increasing interest in the study of the normalized Laplacian as many measures for random walk on binary networks. The eigenvalues and eigenvectors of normalized Laplacian matrix of the associated network are related to the hitting time, mixing time and Kemeny's constant which can be used as a measure of efficiency of navigation on the network [14–17]. Julaiti et al. mentioned that the sum of reciprocals of each nonzero eigenvalues of normalized Laplacian matrix for a network determines the eigentime identity for random walk on the network, which is a global characteristic of the network, and reflects the architecture of the whole network [2]. Zhang et al. presented a first study on the transition weight matrix of a family of weight driven networks. They applied the obtained eigenvalues to derive a closed-form expression for the random target access time for biased random walk occurring on the studied weighted networks [18]. Presently there has been much interest in finding the effective methods to obtain exact expressions for the number of spanning trees of some deterministic networks. Laplacian spectra has often been used for calculating the number of spanning trees [19,20].

Previous works about spectra of the transition matrix were limited to binary networks, and the influence of inhomogeneous weight distribution on the spectral properties of transition matrix still remains unknown. Weighted networks are the extension of binary networks. Diffusion is a key element of a large set of phenomena occurring on natural and social systems modeled in terms of weighted complex networks [21]. Assuming that the diffusion process is local, there are three most general kinds of random walks: random walk, weight-dependent walk and strength-dependent walk. A random walker may choose one of its neighboring edges at the same probability (random walk). In weighted networks, however, the walker will choose an edge according to its weight or the strength of the node connected by it, i.e. weight-dependent walk or strength-dependent walk. Tight-binding quantum gases upon quasiperiodic or fractal-like structures with scale symmetry have been studied intensively over the past few decades [18,22,23]. In most cases, the energy spectrum of the ideal gas and corresponding density of states show self-similarity and power-law behaviors at the same time. Yang et al. studied the thermodynamic behaviors of non-interacting bosons and fermions trapped by a scale-invariant branching structure of adjustable degree of heterogeneity [18].

In this paper, intuited by the weight driven networks [18] and the weighted networks with the weight factor [24–26], the model of the weighted scale-free triangulation networks is built, where the weight factor exhibits some prominent properties that are observed in real-world systems. We study analytically the eigentime identity for the normalized Laplacian matrix of the weighted scale-free triangulation networks for weight-dependent walk. Based on the particular construction of this network, we get the eigenvalues and their corresponding multiplicities for the transition weight matrix of weighted network. Using the obtained eigenvalues for transition weight matrix, we can obtain the eigenvalues for normalized Laplacian matrix. Then, we could deduce an explicit expression for the eigentime identity and its leading scaling shows that the weight factor has an important impact for weight-dependent walk behavior.

2. The weighted scale-free triangulation networks and related quantities

The weighted scale-free triangulation networks, parameterized by a positive number r, is constructed in an iterative manner [27]. We denote the network after t ($t \geq 1$) steps by G_t, which is built as follows.

Initially ($t = 1$), G_1 consists of three nodes and three edges with unit weight forming a triangle. Here the three nodes are denoted by A, B, C.

For $t \geq 2$, G_t is obtained from G_{t-1} by performing the following operation. For each edge with weight w in G_{t-1}, called the original edge, we add one new node linked it to either end of the edge, respectively, and each new edge carries weight rw. Then the operation involved in disconnecting the original edge then adding a new node which is connected with the two original nodes, and the both new edges carries weight w. Fig. 1 illustrates the network generation process from $t = 1$ to 3, Fig. 2 illustrates iterative construction method on every edge. Here we call r the weight factor.

Let N_t, E_t, Q_t denote the total number of nodes, the total number of edges, and the total weight of all edges in G_t, respectively.

For $t \geq 2$, by construction, we have

$$N_t = 3 \cdot 4^{t-1}, \tag{1}$$

$$E_t = 4E_{t-1} = 4^{t-1}E_1 = 3 \cdot 4^{t-1},$$

and

$$Q_t = (2 + 2r)Q_{t-1},$$

which under the initial condition $Q_1 = 3$ yields

$$Q_t = 3(2 + 2r)^{t-1}. \tag{2}$$

For an edge connecting two nodes i and j in G_t, we use $w_{ij}(t)$ to denote its weight. Let $d_i(t)$, $s_i(t)$ denote the degree and strength of node i in G_t, respectively, which is added to the network at generation t_i. It is easy to obtain

$$d_i(t) = 2d_i(t-1) = 2^{t-t_i},$$