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Leonid Berezansky, Alexander Domoshnitsky

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On Stability of the Second Order Neutral Differential Equation

Leonid Berezansky[a], Alexander Domoshnitsky[b]

[a]*Dept. of Math., Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel; e-mail brznsky@cs.bgu.ac.il, phone (972) -50-3047678(corresponding author)*
[b]*Department of Mathematics, Ariel University, Ariel, Israel; e-mail adom@ariel.ac.il*

Abstract

There exists a well-developed stability theory for neutral differential equations of the first order and only a few results on functional differential equations of the second order. One of the aims of this paper is to fill this gap. Explicit tests for stability of linear neutral delay differential equations of the second order are obtained.

Keywords: stability, second order neutral delay differential equations, a priory estimation, Bohl-Perron theorem
AMS Subject Classification: 34K40, 34K20, 34K06

1. Introduction

Neutral differential equations have many applications in control theory, ecology, biology, physics, see, for example, [9, 11, 13, 14, 15]. In particular second order neutral differential equations appear in biology in explaining self-balancing of the human body and in robotics in constructing biped robots (see [10], [12]).

The aim of the present paper is to obtain new explicit exponential stability conditions for the equation

$$\ddot{x}(t)+a(t)\dot{x}(g(t))+b(t)x(h(t))=\sum_{k=1}^{m}a_k(t)x(h_k(t))+\sum_{k=1}^{n}b_k(t)\dot{x}(g_k(t))+\sum_{k=1}^{l}c_k(t)\ddot{x}(r_k(t)). \tag{1.1}$$

Papers [5, 7, 16, 17] are devoted to some asymptotic properties of partial cases of (1.1). In [16] an asymptotic behavior of solutions are studied using analysis of a generalized characteristic equation. In [7] the authors obtain asymptotic formulas for solutions by spectral projection method and an ordinary differential equation method approach. In both papers the equations are considered with variable coefficients and constant delays. In [5, 17] explicit asymptotic stability conditions were obtained for second order autonomous neutral equations using analysis of the roots of quasi-polynomials.

In the paper [6] explicit stability conditions for neutral systems of first order were obtained, but that results cannot be used for stability analysis of equation (1.1).

To obtain new stability tests, we apply the method based on the Bohl-Perron theorem together with a priori estimations of solutions, integral inequalities for fundamental functions of linear delay equations and various transformations of a given equation. We consider equation (1.1) in more general assumptions than in the mentioned before papers: all coefficients and delays are measurable functions, the first derivative of a solution is an absolutely continuous function.

2. Preliminaries

We consider scalar delay differential equation (1.1) under the following conditions:

(a1) $a, b, a_k, b_k, c_k, g, h, g_k, h_k, r_k$ are Lebesgue measurable, a, b, a_k, b_k, c_k are essentially bounded on $[0,\infty)$ functions;

(a2) $0 \le a_0 \le a(t) \le A_0, 0 < b_0 \le b(t) \le B_0, \sum_{k=1}^{l}|c_k(t)| \le C_0 < 1$ for all $t \ge t_0 \ge 0$ and some fixed $t_0 \ge 0$;

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