Applied Mathematics Letters 88 (2019) 96–102

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

Strong competition model with non-uniform dispersal in a heterogeneous environment

Wonhyung Choi[a], Inkyung Ahn[b,*]

[a] Department of Mathematics, Korea University, 145, Anam-ro, Seoul, 02841, Republic of Korea
[b] Department of Mathematics, Korea University, 2511, Sejong-Ro, Sejong, 30019, Republic of Korea

ARTICLE INFO

Article history:
Received 25 May 2018
Received in revised form 18 August 2018
Accepted 18 August 2018
Available online 27 August 2018

Keywords:
Strong competition model
Starvation driven diffusion
Local stability
Mobility ratio

ABSTRACT

In this study, a strong competition model was considered between two species in a heterogeneous environment. For a system with two different constant diffusion rates for each competitor, the fast diffuser can be selected evolutionally under suitable assumptions if the competing interaction between the species is strong. We also claim that a strongly interacting competition leads to a more evolutionary selection than that with the same population dynamics if a species moves with a certain non-uniform dispersal. Furthermore, species with a certain non-uniform dispersal have a competitive advantage over linear random diffusers. In addition, a species with highly sensitive dispersal response to the environment may survive. These strongly competitive advantages were demonstrated by investigating the stability of semi-trivial solutions of the system with non-uniform dispersal and comparing it to the conditions of the model with constant diffusion.

1. Introduction

In this study, we considered the following parabolic system :

$$\begin{cases} u_t = \Delta(\gamma(s)u) + u\,(m(x) - u - av) & \\ v_t = d\Delta v + v\,(m(x) - bu - v) & \text{in } \Omega \times (0,\infty), \\ \nabla(\gamma(s)u)\cdot\vec{n} = \nabla v \cdot \vec{n} = 0 & \text{on } \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x) \ge 0, v(x,0) = v_0(x) \ge 0 & \text{in } \Omega, \end{cases} \tag{1.1}$$

where $u(x,t)$ and $v(x,t)$ represent the densities of the two species at location x and time t. System (1.1) describes two strongly competing species with dispersal, where

$$s = \frac{u + av}{m},$$

* Corresponding author.
E-mail addresses: whchoi@korea.ac.kr (W. Choi), ahnik@korea.ac.kr (I. Ahn).

https://doi.org/10.1016/j.aml.2018.08.014
0893-9659/

and the function $m(x)$ is a positive bounded smooth function, which represents the spatially varying resource productivity affecting both the resource rate and the carrying capacity. Habitat Ω is a bounded region in $\mathbf{R}^N$ with smooth boundary $\partial\Omega$, and $\vec{n}$ denotes the unit normal vector on the boundary.

First, we consider case $\gamma(s) = d_1$, $d = d_2$, where d_1, d_2 are positive constants. We denote such a system by $(1.1)_{const}$. As the following scalar equation

$$\begin{cases} d\Delta u + u(m(x) - u) = 0 & \text{in } \Omega, \\ \nabla u \cdot \vec{n} = 0 & \text{on } \partial\Omega \end{cases} \tag{1.2}$$

has a unique positive solution, denoted by θ_d, $(1.1)_{const}$ provides two semi-trivial solutions $(\theta_{d_1}, 0)$ and $(0, \theta_{d_2})$. Generally, for two competing species with different constant diffusion rates, the slow diffuser has a survival advantage over the fast diffuser in a spatially heterogeneous and temporally invariant environment when the population dynamics for two phenotypes is identical. For example, in [1], the authors showed that in the case $a = b = 1$, if $d_1 < d_2$, then $(\theta_{d_1}, 0)$ is globally asymptotically stable for all non-negative, non-trivial initial data. In this study, it is first claimed that this is not always true under a certain situation for an interacting system with strong competition when $a > 1$ and $b > 1$. For a system with two different constant diffusion rates for each competitor, the fast diffuser can be selected evolutionally if the competing interaction between the two species is strong. More precisely, we have

Theorem 1.1. *Let* $c_* = \sup_{x\in\bar{\Omega}} \frac{m(x)}{\theta_{d_1}(x)} > 1$, *where* $\theta_{d_1}(x)$ *is a unique positive solution when species* v *is absent in* $(1.1)_{const}$.
(i) If $b > c_*$, *then* $(\theta_{d_1}, 0)$ *is linearly stable.*
(ii) If $b < c_*$, *then there exists* $0 < w_{1,d_1} < d_1$ *such that if* $d_2 < w_{1,d_1}$, *then* $(\theta_{d_1}, 0)$ *is linearly unstable, and if* $d_2 > w_{1,d_1}$, *then* $(\theta_{d_1}, 0)$ *is linearly stable.*

Note that the symmetric result of Theorem 1.1(ii) can be obtained for the semi-trivial solution $(0, \theta_{d_2})$. Thus, a threshold number, say w_{1,d_2}, can be found that determines the stability of $(0, \theta_{d_2})$ when $a < c_* = \sup_{x\in\bar{\Omega}} \frac{m(x)}{\theta_{d_2}}$.

However, if a dispersal follows a strategy with a fitness property, the size of the dispersal is not crucial. For instance, starvation-driven diffusion (SDD), which was introduced by Cho and Kim [2], is a dispersal strategy that increases the motility of biological organisms when they are in an unfavorable environment. In competition models with SDD for two phenotypes of a species, Kim et al. [3] demonstrated that the species with SDD has a survival advantage over the species with constant diffusion. In the present study, we claim that for strongly competing systems $(a, b > 1)$, if one competitor follows a dispersal strategy with SDD, then the species has a *strong* competitive advantage, i.e., the species that follows SDD under strongly competing interaction can be selected more evolutionally than that obeying SDD under the same population dynamics $(a = b = 1)$. Furthermore, when the competition is strong, the corresponding interaction can lead to the extinction of the other species, regardless of their diffusion type. In addition, a species with highly sensitive dispersal response to the environment can survive, regardless of dispersal. These strongly competitive advantages are demonstrated by investigating the stability of semi-trivial solutions of the system with SDD, and then comparing it to the conditions of the model with constant diffusion. To this end, system (1.1) where γ is a non-constant function is considered. In the present model, the motility function $\gamma^0(s)$ is defined by

$$\gamma^0(s) = \begin{cases} \ell & \text{for } 0 < s < 1 \\ h & \text{for } 1 < s < \infty \end{cases}$$

which is an increasing function, where $0 < \ell < h$ are constants, and the variable s is the satisfaction measure on the environment defined by $s = \frac{u+av}{m}$. This discontinuous function γ^0 is approximated by a smooth motility function defined by a convolution,

$$\gamma^\varepsilon := \gamma^0 * \eta^\varepsilon,$$