Accepted Manuscript

Positive solution for a class of nonlocal elliptic equations

Huiqin Lu, Xingqiu Zhang

PII: S0893-9659(18)30306-9
DOI: https://doi.org/10.1016/j.aml.2018.08.019
Reference: AML 5630

To appear in: *Applied Mathematics Letters*

Received date : 25 May 2018
Revised date : 22 August 2018
Accepted date : 22 August 2018

Positive solution for a class of nonlocal elliptic equations

Huiqin Lu[a], Xingqiu Zhang[b*]

[a] School of Mathematics Statistics, Shandong Normal University, Jinan, Shandong 250014, PR China

[b] School of Medical Information Engineering, Jining Medical College, Rizhao, Shandong 276826, PR China

Abstract In this paper, we devote ourselves to investigating the existence of positive solution for a class of nonlocal elliptic equations. Our approach is based on the fixed point index theory.

Keywords: Nonlocal elliptic equation; Positive solution; Fixed point index theory

AMS subject classifications (2000): 35J60, 35S15, 37C25, 47G20

1 Introduction

This paper is mainly concerned with the existence of positive solution for the following nonlocal elliptic equations

$$\begin{cases} \mathcal{L}_K u + f(x,u) = 0 & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \backslash \Omega, \end{cases} \tag{1.1}$$

where $\mathcal{L}_K$ is the nonlocal operator defined as follows :

$$\mathcal{L}_K u(x) = \frac{1}{2}\int_{\mathbb{R}^N} (u(x+y) + u(x-y) - 2u(x))K(y)dy, \qquad x \in \mathbb{R}^N, \tag{1.2}$$

here $K: \mathbb{R}^N\backslash\{0\} \longrightarrow (0,+\infty)$ is a function with the properties that

(K_1) $mK \in L^1(\mathbb{R}^N)$, where $m(x) = \min\{|x|^2, 1\}$;

(K_2) there exist $\gamma > 0$ and $s \in (0,1)$ such that $K(x) \geq \gamma |x|^{-(N+2s)}$ for any $x \in \mathbb{R}^N \setminus \{0\}$;

(K_3) $K(x) = K(-x)$ for any $x \in \mathbb{R}^N \setminus \{0\}$.

And the nonlinear term f satisfies the following conditions:

(f_1) $f: \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ is a Carathédory function, $f(x,t)t \geq 0$ for any $x \in \overline{\Omega}$, $t \geq 0$ and $f(x,t) \equiv 0$ for any $x \in \overline{\Omega}$, $t < 0$;

(f_2) there exist $C > 0$ and $q \in (2, 2^*)$ with $2^* = \frac{2N}{N-2s}$ such that $|f(x,t)| \leq C(1 + |t|^{q-1})$;

(f_3) $\lim\limits_{t\to 0^+} \frac{f(x,t)}{t} = f_0$ and $\lim\limits_{t\to\infty} \frac{f(x,t)}{t} = f_\infty$ uniformly for $x \in \overline{\Omega}$.

A typical example for K is that $K(x) = |x|^{-(N+2s)}$, that is when the operator $\mathcal{L}_K$ coincides with the fractional Laplace operator $(-\triangle)^s$ which is nonlocal, different in regularity, Maximum principle from the classical Laplacian operator.

*Corresponding author. E-mail addresses: lhy0625@163.com (H. Lu), zhxq197508@163.com (X. Zhang), Tel. +86 155 88732167