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Positive solution for a class of nonlocal elliptic equations

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Abstract In this paper, we devote ourselves to investigating the existence of positive solution for a class of nonlocal elliptic equations. Our approach is based on the fixed point index theory.

Keywords: Nonlocal elliptic equation; Positive solution; Fixed point index theory

AMS subject classifications (2000): 35J60, 35S15, 37C25, 47G20

1 Introduction

This paper is mainly concerned with the existence of positive solution for the following nonlocal elliptic equations

$$\begin{cases} \mathcal{L}_K u + f(x, u) = 0 & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \quad (1.1)$$

where \mathcal{L}_K is the nonlocal operator defined as follows :

$$\mathcal{L}_K u(x) = \frac{1}{2} \int_{\mathbb{R}^N} (u(x+y) + u(x-y) - 2u(x))K(y)dy, \quad x \in \mathbb{R}^N, \quad (1.2)$$

here $K : \mathbb{R}^N \setminus \{0\} \rightarrow (0, +\infty)$ is a function with the properties that

(K₁) $mK \in L^1(\mathbb{R}^N)$, where $m(x) = \min\{|x|^2, 1\}$;

(K₂) there exist $\gamma > 0$ and $s \in (0, 1)$ such that $K(x) \geq \gamma|x|^{-(N+2s)}$ for any $x \in \mathbb{R}^N \setminus \{0\}$;

(K₃) $K(x) = K(-x)$ for any $x \in \mathbb{R}^N \setminus \{0\}$.

And the nonlinear term f satisfies the following conditions:

(f₁) $f : \overline{\Omega} \times \mathbb{R} \rightarrow \mathbb{R}$ is a Carathéodory function, $f(x, t) \geq 0$ for any $x \in \overline{\Omega}$, $t \geq 0$ and $f(x, t) \equiv 0$ for any $x \in \overline{\Omega}$, $t < 0$;

(f₂) there exist $C > 0$ and $q \in (2, 2^*)$ with $2^* = \frac{2N}{N-2s}$ such that $|f(x, t)| \leq C(1 + |t|^{q-1})$;

(f₃) $\lim_{t \rightarrow 0^+} \frac{f(x, t)}{t} = f_0$ and $\lim_{t \rightarrow \infty} \frac{f(x, t)}{t} = f_\infty$ uniformly for $x \in \overline{\Omega}$.

A typical example for K is that $K(x) = |x|^{-(N+2s)}$, that is when the operator \mathcal{L}_K coincides with the fractional Laplace operator $(-\Delta)^s$ which is nonlocal, different in regularity, Maximum principle from the classical Laplacian operator.

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