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Two methods for estimation of temperature-dependent thermal conductivity based on constant element approximation

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ABSTRACT

Two methods for the estimation of temperature-dependent thermal conductivity are developed. The concept of constant element approximation is introduced, which approximates the thermal conductivity dependence on temperature (*k-T* function) with a step function. A 1-D heat conduction process in a semi-infinite region is considered for the design of the two methods, since an analytical solution describing this process which utilizes the constant element approximation can be found in the literature. The problem concerning the computation of the analytical solution is first solved, and then the analytical solution is applied to develop the two methods. For method I, the surface flux and the movements of the isotherms are recorded. A group of implicit recurrence formulas are established, and the thermal conductivity for each constant element can be determined sequentially in a non-iterative way. For method II, time-varying temperatures at two depths are measured. The thermal conductivities for the constant elements are determined through an optimization process using the Levenberg-Marquardt method. The application of the analytical solution greatly reduces the computational effort spent on the solution process of the inverse problem. Computational examples are presented. The two methods are applied to estimate five types of temperature-dependent thermal conductivities, and the accuracies of the estimated results are discussed. The two methods are proven to be applicable for arbitrary types of *k-T* function, and a prior knowledge concerning the form of the *k-T* function is not necessary.

1. Introduction

Information of thermal conductivity is required for engineering applications associated with the heat transfer process. Many methods have been invented to measure the thermal conductivity including those based on the analysis of inverse heat conduction problem (IHCP) [1,2]. Most of early methods are designed for the situation with constant thermal conductivity, and they are not suitable for the situation with temperature-dependent thermal conductivity. Over the past few decades, significant improvements have been achieved for material sciences, and materials with temperature-dependent thermal conductivities are now frequently encountered. Due to the requirement in engineering applications, methods for estimation of the thermal conductivity dependence on temperature (*k-T* function) have also been developed.

Methods for estimation of the temperature-dependent thermal conductivity can be classified into the steady-state method and the transient method. For the steady-state method, the temperature or the flux is measured at the steady state. The Kirchhoff transform, which is defined as an integral of the *k-T* function with respect to the temperature, is usually applied. The steady-state heat diffusion equation (nonlinear) can be linearized to the Laplace equation using the Kirchhoff transform. The distribution of the Kirchhoff heat function is obtained from the measured data, and then the *k-T* function can be determined from the definition of the Kirchhoff transform. For the transient method, the temperature or the flux is measured during the transient state. The method usually transforms the estimation problem to an optimization problem. The objective of the optimization problem is to minimize the objective function, which is defined as the sum of the squared difference between the calculated results and the measured data. The direct problem and the inverse problem are two basic problems that must be solved during the optimization process.

In most situations, the thermal conductivity can be approximated as a linear function of temperature, and many researchers adopted this

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simplification. Kim et al. [3] considered a conduction process of rod with one end heated and the other insulated. They presented a transient method for estimation of the thermal conductivity from the measured temperatures at both ends. The direct problem was solved by the integral approach with the temperature profile approximated by a third-order polynomial, while the inverse problem was studied using a modified Levenberg-Marquardt algorithm. Czel et al. [4] developed a transient method using the temperature measurements from a 1-D conduction process in the cylindrical coordinate. They adopted the finite difference method for the direct problem and a genetic algorithm for the inverse problem. Mohebbi et al. [5] considered a 2-D conduction process and introduced a steady-state method. Their method did not apply the Kirchhoff transform but adopted an optimization procedure similar to that in the transient method. A body fitted grid generation technique was used for the steady-state direct problem, while the conjugate gradient method was applied for the inverse problem. Ngo et al. [6] investigated a 1-D conduction process of the brass rod heated near the end region. They took into account the convection at the rod surface and established a physical model for the direct problem. The temperatures at several points in the rod were measured, and the thermal conductivity was estimated from the inverse analysis using the Broyden–Fletcher–Goldfarb–Shanno (BFGS) method. Zhang et al. [7] studied a 1-D conduction process of alloys and developed a transient method. The novelty of their work is that the measurement of the temperature was conducted using an infrared line camera. Since the positions of the computational nodes for the direct problem can be set as the positions of the temperature sensors in the camera, the optimization process is greatly simplified.

Besides the linear-type thermal conductivity, the *k-T* function is often described by a linear combination of basic functions with unknown coefficients such as the polynomial. Then the estimation of the *k-T* function becomes the estimation of the unknown coefficients. Yang [8] considered a 1-D conduction process of slab and developed a linear transient method. He established a linear relation between the coefficients of basic functions and the temperatures at the computational nodes after the discretization of the heat diffusion equation. These coefficients can then be determined in a non-iterative way once the time-varying temperatures at the computational nodes are measured. Yang [9,10] also considered a 1-D conduction process with constant boundary fluxes and presented a transient method using the measured temperatures at the two boundaries. The finite difference method was adopted for the direct problem, and the Gauss method was used to solve the inverse problem. Kim [11] presented a steady-state method from the analysis of a 1-D slab conduction process using the Kirchhoff transform. The coefficients of basic functions were related to the data imposed and measured at the two ends. After enough sets of steady-state tests are conducted, these coefficients can then be determined from a group of closed-form equations. Mierzwiczak et al. [12] developed a steady-state method from the analysis of a 2-D conduction process using the Kichhoff transform, in which the measurements of heat flux through the sample and the temperatures at boundaries and some interior points are required. The method of fundamental solutions (MFS) was used to construct the general solution for the linear Laplace equation. The coefficients in the general solution and the coefficients in basic functions were determined together from the measured data also in a non-iterative way. He et al. [13] transformed the problem for the estimation of the coefficients to a multi-objective optimization problem, and solved the problem using the Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II).

Aforementioned methods belong to the category of parameter estimation approach. A prior knowledge regarding the form of the *k-T* function is required so that basic functions can be chosen appropriately and certain accuracy of the estimation can be achieved. There are also situations that no prior knowledge is given concerning the *k-T* function, and then the estimation of the thermal conductivity must be solved using certain function estimation approach. Martin et al. [14] developed a method from the analysis of a steady-state conduction process using the Kirchoff transform. The method requires the measurement of the heat fluxes for all boundary elements on the surface, and also needs the measurement of several boundary temperatures. Kim et al. [15] reconsidered the conduction process presented in Kim et al. [3] and developed a non-iterative transient method for the estimation of *k-T* function. They still adopted a third-order polynomial for the temperature profile, and established a simple relation linking the thermal conductivity with some known parameters. The applicability of their method depends on the accuracy of the polynomial approximation of the temperature profile. Borukhov et al. [16] presented a functional identification approach for the estimation of *k-T* function from the analysis of a 1-D transient conduction process. Continuous temperature measurements were assumed; thus, the objective function was defined as the integral of the squared difference between the calculation and the measurement. They considered three function spaces and defined three types of norms respectively. The conjugate gradient method was adopted to minimize the objective function and to find the optimal *k-T* function in each function space. For the function estimation approach, the computation of the sensitivity matrix is especially time-consuming; Cui et al. [17] adopted a complex-variable differentiation method for the calculation of the sensitivity matrix and developed a transient method from the analysis of a 1-D conduction process. Their method is essentially based on the linear element approximation of the *k-T* function [18]. However, they only presented a simple computational example for the case with a piecewise linear *k-T* function, and a prior knowledge concerning the number of the linear segments was still used during the estimation process.

In this paper, the estimation of temperature-dependent thermal conductivity with no prior knowledge concerning the form of the *k-T* function is investigated, and two function estimation methods based on the constant element approximation of the *k-T* function are developed. Different from existing function estimation methods, an analytical solution is applied for the design of the two methods. On one hand, the analytical solution provides useful formulas that can be applied to develop simple non-iterative methods for the estimation of *k-T* function; on the other hand, the analytical solution makes the solution process of the IHCP much easier and reduces the computational efforts greatly.

The rest of the paper is organized as follows. Section 2 introduces the concept of constant element approximation and presents governing equations (both the original and the transformed) for a 1-D nonlinear heat conduction problem. Section 3 introduces Tao's analytical solution [19] for this problem, and resolves the issue concerning the computation of the solution. Section 4 develops a non-iterative method (method I) for the estimation of *k-T* function, while computational examples of method I are shown in section 5. Section 6 introduces an iterative method (method II) for the estimation of *k-T* function based on the Levenberg-Marquardt method, while computational examples of method II are shown in section 7. Discussions of the two methods are presented in section 8, followed by section 9, with some conclusions.

2. Problem description

2.1. Constant element approximation

In order to accommodate different types of *k-T* function, a constant element approximation which is often used for estimation of the time-varying surface flux [18] is adopted here.

Fig. 1 shows a schematic diagram for the constant element approximation. The variation of the thermal conductivity with the temperature in (U, V) is represented by the dashed curve. The constant element approximation divides (U, V) into several small temperature intervals. Since the temperature interval is small, the thermal conductivity (other thermal properties as well) in each interval can be seemed as a constant. The essential nature of this approximation is to replace the *k-T* function with a few constant elements, and it is also an