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A robust phase unwrapping algorithm based on reliability mask and weighted minimum least-squares method

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ABSTRACT

A robust phase unwrapping algorithm based on reliability mask and weighted minimum least-squares method is proposed. The reliability mask is a 0–1 mask generated according to the reliability map of a wrapped phase map, which is calculated based on the second differences of each pixel. This mask not only ensures the reliability of weighting coefficients but also retains the ability of the 0–1 mask to prevent the noise values from influencing the results. The principle and realization of the proposed algorithm are described in detail. Experiments involving both simulated and actual wrapped phase images are performed to verify the feasibility and effectiveness of the proposed algorithm under various noise conditions. The results show that the proposed method can provide a more reliable and more accurate unwrapped phase, which indicates that our algorithm is more suitable for high-precision phase unwrapping of the actual wrapped phases containing various and complex noises.

1. Introduction

Phase unwrapping has a fundamental role in quantitative measurement in many noncontact optical metrologies, such as synthetic aperture radar interferometry, digital holography, fringe projection, and digital speckle pattern interferometry (DSPI) [1-4]. In these optical techniques, the measured quantities are expressed in the form of a two-dimensional wrapped phase map, whose values vary within the range of $(-\pi, \pi]$ as modulo 2π of the true phases. Phase unwrapping is the procedure used to obtain the true continuous phase field by removing the 2π phase jumps between the neighbouring pixels.

In the past 20 years, a number of phase unwrapping algorithms have been proposed, which can be grouped into three classes [5-7] : (1) region algorithms, (2) path-following algorithms, and (3) global algorithms. The most widely used global algorithm used in two-dimensional phase unwrapping is the least-squares method. This method minimizes the differences between the partial derivatives of the wrapped phase values and those of the unwrapped phase values [8]. In 1989, Ghiglia and Romero found that the least-squares solution to the phase unwrapping problem is mathematically identical to the solution of the Poisson equation on a rectangular grid with Neumann boundary conditions [9]. In 1994, Pritt et al. presented an unweighted least-squares unwrapping algorithm based on fast Fourier transformation (FFT) [10]. Subsequently, various solutions for the Poisson equation have been put forth, and the solutions can be divided into direct methods and iterative methods [11]. Direct methods such as FFT and discrete cosine transform are very fast but cannot unwrap inconsistent phases such as shadows and layover [12]. Simple iterative methods such as those of Jacobi or Gauss-Seidel or successive overrelaxation can be applied only to phase unwrapping on a relatively small grid [13]. Weighted iteration methods are not only fast and robust but also able to control the propagation of error arising from the smooth effect by using the weighting coefficient [10]. In general, the weighting coefficient is often determined by quality map, which is calculated with the use of functions such as phase derivative variance (PDV) [14], pseudo-correlation [15], maximum phase gradient [16], and others. In addition, the weighting coefficient is also determined by the 0–1 mask, which assigns a weight of 0 to the inconsistent data and a weight of 1 to the good data [7]. Ghiglia and Romero reported that the 0–1 mask could obtain a correct solution in a region corresponding to unity weights and maintain continuity across a region corresponding to zero-valued weights when additional information allows the definition of a corresponding weighting array [11]. The concern is how to generate a reliable 0–1 mask for wrapped phase with inconsistent data caused by random noise, aliasing, and so on.

In this article, inspired by the fact that the reliability of each pixel is able to detect the inconsistencies of the phase map effectively in path unwrapping algorithms, we present a novel phase unwrapping algorithm based on the reliability mask and weighted minimum least-squares method (RM-WLS). This reliability mask is a 0–1 mask determined according to the reliability of each pixel in the wrapped phase. Therefore, it can ensure the reliability of the 0–1 mask and retain the

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advantages of the 0–1 mask, improving the accuracy of the unwrapping phase and eliminating the effect of the noise.

2. Principle

2.1. Reliability mask

In our proposed RM-WLS phase unwrapping algorithm, the weighting coefficient is derived from a novel 0–1 mask, which is called reliability mask. The reliability mask is determined according to the reliability of each pixel in the path unwrapping algorithms. The reliability of each pixel can be calculated with the use of different reliability functions, which are usually based on the gradients or differences between a pixel and its neighbours. Among the differences, the second difference is able to measure the degree of concavity/convexity of the phase function, so the reliability function based on the second differences provides better detection of possible inconsistencies in the phase map. Therefore, here we adopt the second differences to calculate the reliability map for a $M \times N$ wrapped phase, which is expressed as

$$R_{i,j} = \frac{1}{\sqrt{H^2(i,j) + V^2(i,j) + {D_1}^2(i,j) + {D_2}^2(i,j)}}, \tag{1}$$

where

$$\begin{aligned} H(i,j) &= W\{\psi_{i,j-1} - \psi_{i,j}\} - W\{\psi_{i,j} - \psi_{i,j+1}\} \\ V(i,j) &= W\{\psi_{i-1,j} - \psi_{i,j}\} - W\{\psi_{i,j} - \psi_{i+1,j}\} \\ D_1(i,j) &= W\{\psi_{i-1,j-1} - \psi_{i,j}\} - W\{\psi_{i,j} - \psi_{i+1,j+1}\} \\ D_2(i,j) &= W\{\psi_{i-1,j+1} - \psi_{i,j}\} - W\{\psi_{i,j} - \psi_{i+1,j-1}\} \end{aligned}, \tag{2}$$

where $1 \le i \le M-2$ and $1 \le j \le N-2$, $R_{i,j}$ is the reliability of point (*i, j*); *W* is the wrapping operator that wraps all values of its argument into the range $(-\pi, \pi]$ by adding or subtracting integral multiples of 2π from its argument; *H*(*i, j*) and *V*(*i, j*) represent the horizontal and vertical gradients of wrapped phase $\psi_{i,j}$; and $D_1(i,j)$ and $D_2(i,j)$ represent diagonal gradients of $\psi_{i,j}$. The reliabilities of those points on the borders of $\psi_{i,j}$ are set to 0.

The reliability mask is obtained by thresholding the reliability map, where values below the threshold (lower reliability) are set to 0 and values beyond (higher reliability) are set to 1. It is defined by the following equation:

$$q_{i,j} = \begin{cases} 1, R_{i,j} \ge \theta \\ 0, R_{i,j} < \theta \end{cases}, \tag{3}$$

where θ is the reliability threshold, and $q_{i,j}$ is the value of the point (*i, j*) in the reliability mask. The threshold θ is empirically set in real experiments, while the optimal threshold θ can be found in simulation testing.

2.2. Phase unwrapping algorithm based on the reliability mask

The relationship between the true phase $\phi_{i,j}$ and its wrapped phase $\psi_{i,j}$ on a rectangular $M \times N$ grid can be written as

$$\phi_{i,j} = \psi_{i,j} + 2\pi k_{i,j}, \tag{4}$$

where $k_{i,j}$ is an integer.

The basic idea of the least-squares phase unwrapping algorithm is to minimize the distance between the partial derivative of $\phi_{i,j}$ and the phase difference of $\psi_{i,j}$. The phase differences of the $\psi_{i,j}$ in the vertical and the horizontal direction are defined as

$$\begin{aligned} \Delta^x_{i,j} &= \begin{cases} W\{\psi_{i+1,j} - \psi_{i,j}\}, (i = 0...M-2, j = 0...N-1) \\ 0, otherwise \end{cases} \\ \Delta^y_{i,j} &= \begin{cases} W\{\psi_{i,j+1} - \psi_{i,j}\}, (i = 0...M-1, j = 0...N-2) \\ 0, otherwise \end{cases} \end{aligned}, \tag{5}$$

The least-squares solution is the solution $\phi_{i,j}$ that minimizes *J*

$$J = \sum_{i=0}^{M-2}\sum_{j=0}^{N-1}\left(\phi_{i+1,j} - \phi_{i,j} - \Delta^x_{i,j}\right)^2 + \sum_{i=0}^{M-1}\sum_{j=0}^{N-2}\left(\phi_{i,j+1} - \phi_{i,j} - \Delta^y_{i,j}\right)^2. \tag{6}$$

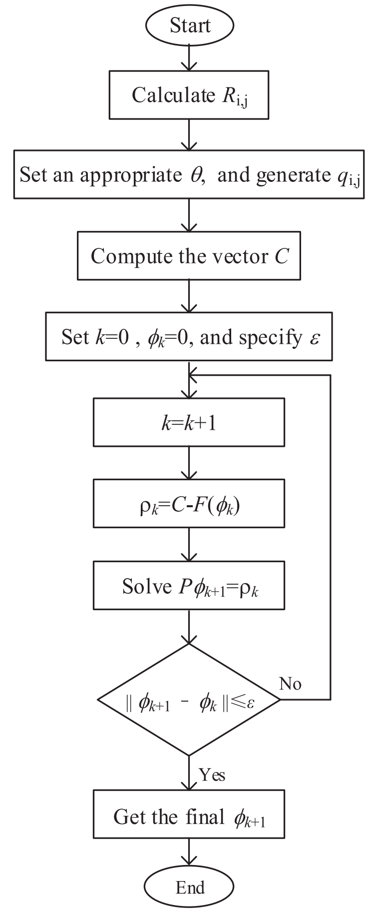

Fig. 1. Framework of the proposed robust RM-WLS algorithm.

According to Hunt's matrix formulation, Eq. (6) is equivalent to solving the following linear equation

$$\left(\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}\right) + \left(\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}\right) = \rho_{i,j}, \tag{7}$$

where

$$\rho_{i,j} = \left(\Delta^x_{i,j} - \Delta^x_{i-1,j}\right) + \left(\Delta^y_{i,j} - \Delta^y_{i,j-1}\right). \tag{8}$$

It is obvious that Eq. (7) is a discretization of the Poisson equation under the Neumann boundary conditions, which can be solved by using direct and iterative least-square phase unwrapping methods.

The proposed RM-WLS algorithm is an iterative least-square phase unwrapping method, and it adopts the Picard iteration. The framework of the new algorithm is shown in Fig. 1. The detailed realization steps are as follows:

1 Step 1: Compute the reliability value of each pixel$R_{i,j}$.
2 Step 2: Set an appropriate reliability threshold θ to determine the 0–1 reliability mask $q_{i,j}$.
3 Step 3: Perform the Picard iteration operation:
   a Compute the vector *C* formed from the modified discrete Laplace operator of the weight wrapped phase differences,

$$\begin{aligned} C_{i,j} &= \min\left(q_{i+1,j}, q_{i,j}\right)\Delta^x_{i,j} - \min\left(q_{i,j}, q_{i-1,j}\right)\Delta^x_{i-1,j} \\ &+ \min\left(q_{i,j+1}, q_{i,j}\right)\Delta^y_{i,j} - \min\left(q_{i,j}, q_{i,j-1}\right)\Delta^y_{i,j-1} \end{aligned} \tag{9}$$

   b Set the iteration counter $k = 0$, the initial true phase $\phi_k = 0$, and specify the iteration threshold ε.
   c Compute the vector $\rho_k = C - F(\phi_k)$,