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Investigating a common premise in structural health monitoring: Are higher modal frequencies more sensitive to an incipient crack on a beam than lower ones?

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A B S T R A C T

This paper has investigated a well-known premise in structural health monitoring that higher modal frequencies are more sensitive to an incipient crack on a beam than lower ones. Representing the fractional reduction in modal frequency caused by the crack, normalized modal frequency shift (NMFS) is formulated and expressed as a simple closed-form solution. The NMFS-factor which determines the NMFS is proved to decline as the mode number increases implying that the fractional reduction in modal frequency becomes less sensitive to the crack in higher frequency range. The validity of the proposed approach is demonstrated through the comparison to the finite element analysis results and experimental data. Theoretical proof, numerical and experimental observations have indicated that the fractional reduction in modal frequency decreases with mode number though the absolute reduction increases.

1. Introduction

Detecting an incipient crack on a beam has been of a great concern for structural health monitoring applications because beams are widely used for engineering structures [1–3]. Advances in sensor technologies allow for simultaneous actuating and sensing of the high-frequency dynamic responses of a beam through smart sensors such as piezoelectric (PZT) wafers and macro fiber composite (MFC) [1,2,4]. In the context of elastodynamics, the dynamic responses of a beam can be classified into two categories, that is, wave and vibration. The basic premise for enabling crack detection of a beam is that both wave and vibration responses alter at the presence of the crack when compared to those obtained from the intact status of the beam [5].

In case of wave response, wave reflection and mode conversion occur when incident wave encounters a crack. The pulse-echo method is one of the most popular crack detection schemes using the wave reflection due to a crack. The existence of the crack can be confirmed through the reflection coefficient being the ratio of the amplitude of reflected wave to that of incident wave [2]. It has been demonstrated both numerically and experimentally that the reflection coefficient increases proportionally to the frequency of excitation because incident wave with shorter wavelength tends to induce larger reflected wave at the crack [6]. Therefore, high-frequency wave responses are needed to improve the detection capability of an incipient crack.

In case of vibration response, a transverse crack reduces the flexural stiffness of a beam while the mass of the beam rarely changes. As a result, the modal frequencies of cracked beams decrease compared to those of an intact beam [3]. Concerning crack detection capability, the aforementioned concept in wave response has been extended to the vibration response realm. In other words, higher-modal frequency become more sensitive to the incipient crack of a beam because the size of the crack that can be detected from changes in system dynamics is inversely proportional to the frequency range of excitation [5]. Here, the underlying mechanism is that there exists elastodynamic equivalence between wavelength in wave response and modal wavelength in vibration response from the viewpoints of crack detection capability. Under this mechanism, a common premise has been established that higher modal frequencies are more sensitive to an incipient crack on a beam than lower ones [1,3,4,5]. However, there has been no rigorous theoretical study to demonstrate the validity of the premise.

In this regard, this paper aims to investigate the validity of the premise through both theoretical and experimental study regarding a beam with an incipient crack. First, the elastodynamic behavior of the cracked beam is represented based on the Timoshenko beam theory and the local flexibility of the crack [7]. Then the sensitivities of high-frequency wave and vibration to an incipient crack are formulated and expressed as closed-form solutions of the reflection coefficient and the normalized modal frequency shift (NMFS), respectively. Here, the

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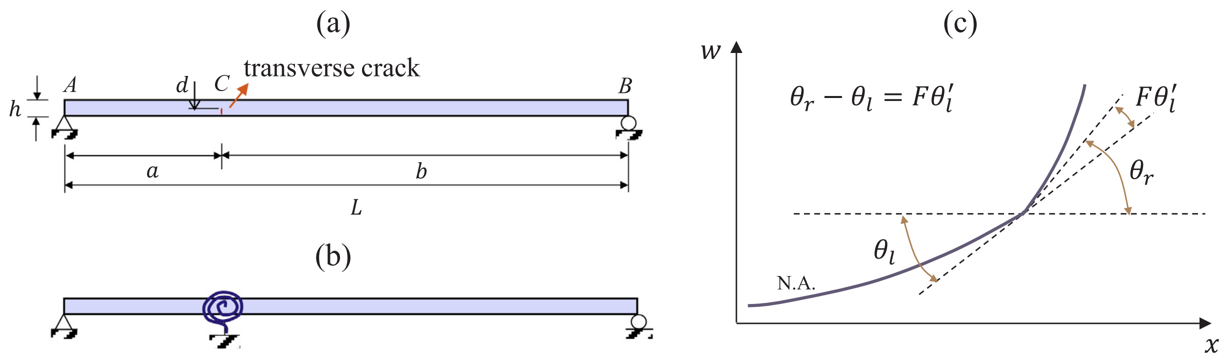

Fig. 1. (a) A simply-supported beam with a transverse crack; (b) A beam with a spring representing the local flexibility of a crack; (c) Compatibility of a rotational angle at crack location.

NMFS denotes the modal frequency shift of a cracked beam which is normalized by the corresponding modal frequency of an intact beam. Note that the NMFS represents the fractional reduction of modal frequency caused by the crack which has been widely studied in previous works regarding dynamic analysis and damage assessment of a cracked beam [8–10]. The NMFS also can play a role as a counterpart to the reflection coefficient because both are dimensionless values ranging from 0 to 1 depending on the crack depth. Not only the closed-form solutions of the reflection coefficient and the NMFS describe the sensitivities of high-frequency wave and vibration responses to an incipient crack on beam, but also reveal the fundamental difference between them explicitly. Finally, the closed-form solution of the NMFS is validated through the comparison to finite element analysis and experimental data regarding a free-free aluminum beam with a center crack.

2. Spectral representation of the dynamic behavior of a beam with an incipient crack

2.1. Assumptions

Fig. 1(a) presents a simple beam with a transverse edge crack. Four assumptions are introduced to justify the theoretical formulation: (i) The crack remains open during the dynamic behaviors of a beam, (ii) The beam follows Timoshenko beam theory in which the effects of shear deformation and rotational inertia are taken into account, (iii) The frequency range is less than the cut-off frequency meaning that there exist one propagating flexural wave and one evanescent wave mode [11], and (iv) The mode conversion from the flexural to the axial wave mode and vice versa is not considered.

Under these assumptions, the crack can be represented by a spring with local flexibility as shown in Fig. 1(b) to predict the dynamic behaviors of the beam [7]. Local flexibility F determines the compatibility of a rotational angle at crack location as shown in Fig. 1(c):

$$\theta_r - \theta_l = F\theta_l' \tag{1}$$

where θ_l and θ_r denote rotational angles of the neutral axis of the beam at the left and the right side of the crack, respectively while θ_l' represents the spatial derivative of θ_l with respect to x. Eq. (1) indicates that the discontinuity of the rotational angle at the crack is proportional to the local flexibility (F) and the curvature (θ_l') of the beam at the left side of the crack. F depends on the material properties and the geometry of the beam, and crack depth ratio ($\bar{\delta} = \frac{d}{h}$). In this paper, the cross-section of the beam is assumed to be rectangular and F is approximately expressed as follows in case of an incipient crack [12]:

$$F = 6\pi h\bar{\delta}^2\left\{\frac{1.12^2}{2}(1-\nu^2)\right\} \tag{2}$$

where ν denotes Poisson ratio of the beam.

2.2. Spectral representation of a Timoshenko beam with a crack

The dynamic behavior of a Timoshenko beam is governed by the dispersion equation which expresses relations between wave number k and angular frequency ω [13]:

$$k^4-\omega^2\left(\frac{1}{c_0^2}+\frac{1}{c_Q^2}\right)k^2+\left(\frac{\omega^4}{c_0^2c_Q^2}-\frac{\omega^2}{c_0^2r^2}\right)=0 \tag{3}$$

$$c_0^2=\frac{E}{\rho},\quad c_Q^2=\frac{\kappa G}{\rho},\quad r^2=\frac{I}{A}$$

where E, G, κ, ρ, I, and A represent the elastic modulus, the shear modulus, the shear coefficient, the density, the second moment of inertia, and the cross-section of a beam, respectively. In case ω is less than the cut-off frequency ($\omega_{\text{cut-off}} = \frac{c_Q}{r}$), wave numbers k_p and k_e are expressed with respect to ω as follows through solving Eq. (3):

$$k_p=\sqrt{\frac{X+\sqrt{X^2-4Y}}{2}},\ k_e=\sqrt{\frac{-X+\sqrt{X^2-4Y}}{2}} \tag{4}$$

$$X=\omega^2\left(\frac{1}{c_0^2}+\frac{1}{c_Q^2}\right),\ Y=\frac{\omega^4}{c_0^2c_Q^2}-\frac{\omega^2}{c_0^2r^2}$$

Note that k_p and k_e are responsible for propagating wave mode and evanescent wave mode, respectively.

Using these wave numbers, the spectral displacements and rotational angles of the cracked Timoshenko beam in Fig. 2 can be divided into two parts at the left and right sides of the crack, respectively [14]:

$$w_l = A_1e^{ik_px_l}+A_2e^{-ik_px_l}+A_3e^{k_ex_l}+A_4e^{-k_ex_l}$$
$$w_r = B_1e^{-ik_px_r}+B_2e^{ik_px_r}+B_3e^{-k_ex_r}+B_4e^{k_ex_r} \tag{5a}$$

$$\theta_l = -iPA_1e^{ik_px_l}+iPA_2e^{-ik_px_l}-NA_3e^{k_ex_l}+NA_4e^{-k_ex_l}$$
$$\theta_r = -iPB_1e^{-ik_px_r}+iPB_2e^{ik_px_r}-NB_3e^{-k_ex_r}+NB_4e^{k_ex_r} \tag{5b}$$

where w_l and w_r denote transverse displacements of the neutral axis of the beam at the left and the right side of the crack in Fig. 2,

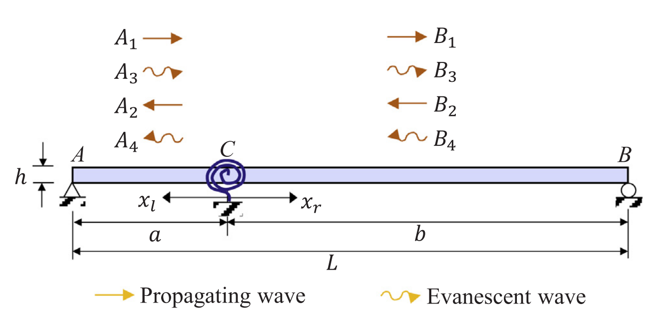

Fig. 2. Unknowns and a dual coordinate system for the dynamic behavior of a beam with a flexibility spring representing a crack.