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On a new methodology for ranking fuzzy numbers and its application to real economic data

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Abstract

Ranking fuzzy numbers plays an important role in the decision process. Several strategies can be found in the abundant literature existing on this topic, which can be basically classified into two main groups: approaches based on the so-called ranking indices and based on fuzzy binary relations. Ranking indices are functions from fuzzy numbers to real values. These defuzzification methods are critiqued by its implicit information loss which leads to results that might not be consistent with human intuition in certain cases. In a fuzzy context, approaches based on binary relations can be considered more suitable solutions. Following this strategy, this paper introduces a new technique for ranking fuzzy numbers which is applicable to the whole set of fuzzy numbers. It is proved that many desired properties (in particular, we give several new properties) are satisfied and numerical examples show reasonable results. The developed technique is very easy to compute and interpret in practice, and it overcomes certain shortcomings that appear when applying other more complex algorithms. In the case of triangular or trapezoidal fuzzy numbers, the procedure is particularly simple and intuitive. Finally, a real-life example involving data of Chinese Economy is also carried out.

Keywords: Fuzzy number; Ranking; Shibor; Decision making; Fuzzy set

1. Introduction

One of the most interesting open problems in fuzzy sets theory is to find an appropriate way for ranking fuzzy numbers. These methods have many applications in different fields such us decision theory, artificial intelligence, financial and socioeconomic assessment, optimization problems, etc. For example, a procedure for ranking fuzzy numbers with appropriate properties would be applied to fuzzy decision making, which is a very attractive discipline because of their applications in Economics and Business. Other fields of study such as optimization, approximate reasoning, fuzzy control and financial and socioeconomic assessment can also take advantage of a methodology for

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ranking fuzzy numbers that produce reasonable results. In fuzzy regression theory, a partial order in the set of all fuzzy numbers would help to introduce a fuzzy methodology for obtaining optimal solutions ([3,26,28]), that is, it would be very useful to introduce a partial order on the data set in order to find the model that reaches the lowest fuzzy error.

Due to the special characteristics of fuzzy numbers, there is not a universal and globally accepted way to rank fuzzy numbers. After the first approach to this problem, presented by Jain [19] in 1976, many ranking methods have been introduced in the last decades from different points of view (see, for instance, [8,20]). Very recently, Ban and Coroianu [6] introduced an extensive review on the current status this problem. They basically distinguished two approaches:

- Based on the so-called ranking indices, which are functions from fuzzy numbers to real values.
- Based on fuzzy binary relations.

The choice of a ranking method will have a remarkable effect in the solution of the problem and based in the results obtained from a few numerical examples we cannot conclude what approach is better. A ranking fuzzy approach will be considered a good choice if some desired properties are preserved. Wang and Kerre [31] proposed a list of reasonable properties for the ordering of fuzzy quantities and Ban and Coroianu [6] adapt these requirements for the case when the ordering approach is induced by a binary relation. They can be summarized as follows.

Let $\mathcal{S}$ be a subset of the family of all fuzzy numbers $\mathcal{F}$. Given a binary relation $\preceq$ on $\mathcal{F}$, we write $\mathcal{A} \sim \mathcal{B}$ when $\mathcal{A} \preceq \mathcal{B}$ and $\mathcal{B} \preceq \mathcal{A}$, and we write $\mathcal{A} \prec \mathcal{B}$ if $\mathcal{A} \preceq \mathcal{B}$ holds and $\mathcal{B} \preceq \mathcal{A}$ is false.

($\mathbb{A}_1$) (Reflexivity) $\mathcal{A} \preceq \mathcal{A}$ for all $\mathcal{A} \in \mathcal{S}$.
($\mathbb{A}_2$) For any $\mathcal{A}, \mathcal{B} \in \mathcal{S}$, from $\mathcal{A} \preceq \mathcal{B}$ and $\mathcal{B} \preceq \mathcal{A}$ results $\mathcal{A} \sim \mathcal{B}$.
($\mathbb{A}_3$) (Transitivity) For any $\mathcal{A}, \mathcal{B}, \mathcal{C} \in \mathcal{S}$, from $\mathcal{A} \preceq \mathcal{B}$ and $\mathcal{B} \preceq \mathcal{C}$ results $\mathcal{A} \preceq \mathcal{C}$.
($\mathbb{A}_4$) For any $\mathcal{A}, \mathcal{B} \in \mathcal{S}$, from $\sup \operatorname{supp} \mathcal{A} \leq \inf \operatorname{supp} \mathcal{B}$ results $\mathcal{A} \preceq \mathcal{B}$.
($\mathbb{A}'_4$) For any $\mathcal{A}, \mathcal{B} \in \mathcal{S}$, from $\sup \operatorname{supp} \mathcal{A} < \inf \operatorname{supp} \mathcal{B}$ results $\mathcal{A} \prec \mathcal{B}$.
($\mathbb{A}_5$) If $\mathcal{A}, \mathcal{B}, \mathcal{A}+\mathcal{C}, \mathcal{B}+\mathcal{C} \in \mathcal{S}$ are such that $\mathcal{A} \preceq \mathcal{B}$, then $\mathcal{A}+\mathcal{C} \preceq \mathcal{B}+\mathcal{C}$.
($\mathbb{A}'_5$) If $\mathcal{A}, \mathcal{B}, \mathcal{A}+\mathcal{C}, \mathcal{B}+\mathcal{C} \in \mathcal{S}$ are such that $\mathcal{A} \prec \mathcal{B}$, then $\mathcal{A}+\mathcal{C} \prec \mathcal{B}+\mathcal{C}$.
($\mathbb{A}_6$) If $\mathcal{A}, \mathcal{B} \in \mathcal{S}$ and $\lambda \in \mathbb{R}$ are such that $\lambda\mathcal{A}, \lambda\mathcal{B} \in \mathcal{S}$, from $\mathcal{A} \preceq \mathcal{B}$ follows $\lambda\mathcal{A} \preceq \lambda\mathcal{B}$ if $\lambda \geq 0$, and $\lambda\mathcal{B} \preceq \lambda\mathcal{A}$ if $\lambda \leq 0$.

From our point of view, although all approaches are interesting, those rankings generated by a procedure based on the standard ordering of reals are not consistent with the idea of vagueness/uncertainly which is intrinsic to the fuzzy theory. In other words, if the order between two fuzzy numbers depends on a crisp number (for instance, the area between them), this method may not capture both imprecision and uncertainty and, consequently, it is not consistent with factors such as vagueness and ambiguity which affect the behavior of the phenomenon studied in the fuzzy setting. It would be more reasonable to use genuine fuzzy techniques rather than a real number to ranking fuzzy numbers. Consequently, it is necessary to introduce new ranking procedures satisfying three main characteristics:

- They must not be based in a unique real number (in order to avoid loss of information).
- They must verify as many reasonable properties as possible.
- But, above all, they must be as coherent with human intuition as possible.

Having in mind such constraints, the main aim of this manuscript is to introduce a new fuzzy binary relation in the whole set of fuzzy numbers that can be used in practice to compare two distinct fuzzy numbers. This binary relation is able to lead to coherent with human intuition comparisons. It especially takes into account the geometric shape of the fuzzy numbers and the measurable subsets in which a fuzzy number is clearly less than or equal to another fuzzy number. As its result does not depend on a unique crisp number, this method is not a ranking index. Furthermore, it is compatible with addition and scalar multiplication. In fact, it satisfies the greater number of reasonable properties that can be found in literature (more than analyzed in [19,6]), which causes that it is according to human intuition in most of cases. Fuzzy data analyzed show that the main advantage of the proposed methodology is its capability to provide a correct ordering of generalized trapezoidal fuzzy numbers. We point out that our technique is applicable to the whole set of all fuzzy numbers (not only on trapezoidal fuzzy numbers), although it is particularly simple and easy

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