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Analysis of discontinuous dynamical behavior of a class of friction oscillators with impact☆

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ABSTRACT

In this paper, the discontinuous dynamical behaviors in the friction oscillator with impact are investigated by using the theory of flow switchability in discontinuous dynamical systems. On the basis of discontinuity of friction and impact, different domains and boundaries are defined for such system. Based on above domains and boundaries, the analytical conditions of passable motion, stick motion, sliding motion and grazing motion are obtained mathematically. The analytical predictions of the corresponding periodic motion are given by the adequate mapping structures. The passable, stick, sliding, grazing and periodic motions are illustrated through trajectories in phase planes and time histories of displacement, velocity and G-function for a better understanding of motion mechanism of the friction oscillator with impact. In the end, the grazing and sliding bifurcations are determined by the local singularity theory of discontinuous dynamical systems. This model is suitable for nonlinear dynamical description of the vibration in rotor bearing or gear transmission systems with impact and friction.

1. Introduction

Piecewise linear dynamical systems are widely used to describe engineering vibrations, such as vibration in gear box and rotor-bearing systems. In 1932, Hartog and Mikina [1] investigated the piecewise linear system without damping, and the closed-form solution for periodic symmetric motion was achieved. In 1960, Levitan [2] studied the existence of periodic motions in a periodically forced frictional oscillator. In 1964, Filippov [3] investigated the motion in the Coulomb friction oscillator and presented differential equations with discontinuous right-hand side. To determine the sliding motion along the separation boundary, the differential inclusion was introduced via the set valued analysis, and the existence and uniqueness of solutions for such a discontinuous differential equation were discussed. The detailed discussion of such discontinuous differential equations can be referred to Filippov [4]. Owing to high speed requirement in gear systems, the vibration and noise become very serious problems, and the linear vibration model cannot provide the adequate prediction. In 1984, Pfeiffer [5] developed an impact model to describe regular and chaotic motions in gear box (also see [6]). In 1987, Whiston [7] focused on the global dynamics on differentiable manifolds and local dynamics in Poincare fixed points neighborhoods of a vibro-impact linear oscillator. Furthermore, Balachandran et al. [8–10] used a combination of experiments and numerical simulations to investigate the dynamics of an elastic structure which was excited by harmonic and anharmonic impactor motions. In 2001, Balachandran [11] studied the dynamics and stability of milling operations and cylindrical mills by using an unified mechanics-based model. In 1990, Andreaus [12] investigated the sliding–uplifting response of rigid block to base excitation. For a better understanding of the nonlinear responses in the piecewise linear system, Wong et al. [13] used the incremental harmonic balance method to obtain the periodic motions of unsymmetrical piecewise linear systems in 1991, and Kim and Noah [14,15] gave the stability and bifurcation analysis of periodic motion in 1991 and 1998, respectively. In 1983, Shaw and Holmes [16] investigated a piecewise linear system with a single discontinuity by using mapping techniques. In 1989, Natsiavas [17] perturbed the initial conditions to determine periodic motions. In 1991, Nordmark [18] investigated the non-periodic motion caused by the grazing bifurcation. Andreaus and Casini studied the dynamics of three-rigid block assemblies with unilateral deformable contacts in [19,20] in 1999, and also investigated the dynamics of friction oscillators excited by moving base or/and driving force or colliding with a rigid or deformable obstacle in [21–24] in 2001 and 2002, respectively. Pascal [25–28] studied dynamics and stability of a two degrees of freedom oscillator with an elastic stop, dynamics of coupled oscillators excited

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by dry friction, new events in stick–slip oscillators behavior and a new model of dry friction oscillator colliding with a rigid obstacle. In 2004, Luo and Menon [29] investigated global motions in a periodically forced linear system through mapping structures. The mapping for grazing bifurcation phenomena was developed in [30,31] by Bernardo et al. Therefore, a periodically forced and piecewise linear system with impact is of great interest. Other discontinuous systems are investigated in [32–43].

However, the dynamical behaviors of discontinuous dynamical systems are still difficult to be investigated. In 2005, Luo [44,45] developed a general theory to study discontinuous dynamical systems on connectable domains and introduced the imaginary, sink and source flows, and also developed the necessary and sufficient conditions of sink and source flows. In 2008, Luo [46] defined G-functions and developed a theory to determine the flow switchability to the discontinuous boundary through G-functions. The detailed discussion can be referred to Luo [47–49]. According to the theory of discontinuous dynamical systems, in 2016, Chen and Fan [50] researched the analytical conditions of flow switchability of motions for a double friction-oscillator. In 2017, Fan *et al.* [51] studied dynamical behaviors of a friction-induced oscillator with 2-DOF(degree of freedom) on a speed-varying traveling belt. Based on this theory, lots of discontinuous models have been investigated, for example [52–63].

Discontinuous dynamical systems have been used to study gearboxes and transmission systems. In 1988, Nevzat and Houser [64] introduced a general classification of mathematical models for studying gear dynamics. The basic properties of each model were given and different parameters were selected to discuss gear dynamics models. The dynamic behavior of gear rotors was studied by establishing finite elements of gear rotors in 1990 by Kahraman et al. [65]. In 1994, Choy and Polyshchuk [66] analyzed the effects of surface pitting and wear on a gear transmission system by changing the phase and magnitude of gear meshing stiffness in a gear transmission model. In 1995, Rook and Singh [67] studied the dynamic behavior of a reverse–idler gear pair with concurrent clearances. In 2005, Luo [68] studied the periodic motion and grazing motion of a piecewise linear oscillator driven by harmonic forces through the flow switchability theory in discontinuous dynamical systems; and gave the numerical simulations. In the previous study of dynamical behavior of gear transmission systems, only impact was considered, but friction in actual problems was not considered and some dynamical behaviors were not studied enough.

This paper presents a new class of piecewise linear system model - a friction oscillator with impacts. This model captures both impact and friction behaviors in engineering applications like rotor bearing and gear transmission systems. The theory of flow switchability is then used to investigate the discontinuous dynamical behavior of this model. The rest of the paper is organized in the following manner. In Section 2, the physical model of the friction system with impact is introduced. The absolute phase plane of such system is divided into different domains and boundaries due to the impact and friction discontinuities in Section 3. In Section 4, the G-functions are introduced to illustrate the motion switching mechanism, and the analytical conditions of passable motion, sliding motion, stick motion and grazing flow are presented mathematically. The switching sets and basic mappings are defined for such a friction–impact system to describe the periodic motions with different mapping structures in Section 5. In Section 6, numerical simulations of passable motion, stick motion, grazing flow and periodic motion are carried out to illustrate the analytical conditions of motion switchability for a better understanding of the complex dynamical behaviors of such friction oscillator with impact. The sliding and grazing bifurcation scenarios of varying driving frequency are given in Section 7. Finally, Section 8 concludes the paper.

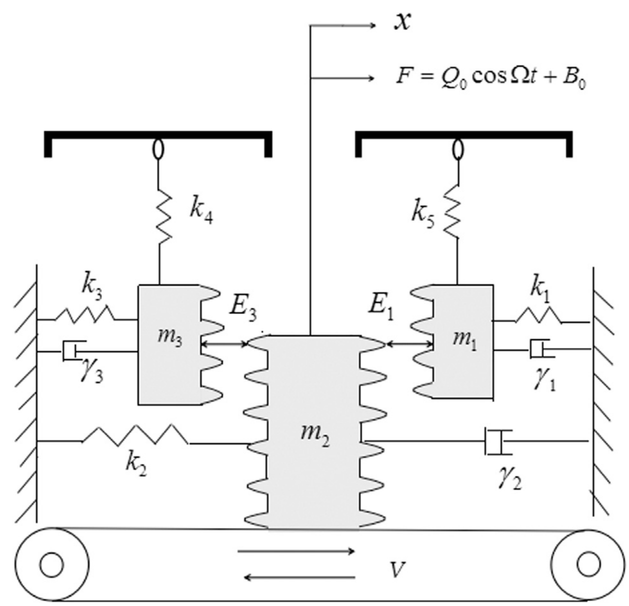

Fig. 1. Physical model.

2. Physical model

Consider a piecewise linear oscillator with impact at two displacement boundaries and friction between the masses and a conveyor belt, as shown in Fig. 1. There are three masses $m_j (j = 1, 2, 3)$ in this system. They are connected to corresponding springs with coefficients $k_j (j = 1, 2, 3)$ and dampers with coefficients $\gamma_j (j = 1, 2, 3)$. To balance gravity of two masses m_3 and m_1, two vertical springs with coefficients k_4 and k_5, which are respectively connected with two masses m_3 and m_1, are hanged on the two sliding brackets. The external force $F = Q_0 \cos \Omega t + B_0$ is applied to the main mass m_2, where Q_0, Ω and B_0 are excitation amplitude, frequency and constant force. And the conveyor belt moves with constant speed V. Through prestressing the mass m_2, the constant-force magnitude of B_0 could be adjusted to adapt to different working environments. We assume that the origin of the system in absolute coordinates is the equilibrium position of the mass m_2, and the position of m_2 is denoted by x, and the distance between the mass $m_3(m_1)$ and the mass m_2 is $E_3(E_1)$. The friction force between the main mass m_2 and the conveyor belt shown in Fig. 2 is described as

$$F_f(\dot{x}) \begin{cases} = \mu_k F_N, & \dot{x} > V, \\ \in [-\mu_k F_N, \mu_k F_N], & \dot{x} = V, \\ = -\mu_k F_N, & \dot{x} < V, \end{cases} \tag{2.1}$$

where $\dot{x} = dx/dt$, μ_k is the frictional coefficient between the main mass m_2 and the belt, $F_N = m_2 g$ is the normal force exerting on the conveyor belt.

To develop simple model for gear transmission systems, the following assumptions are adopted:

(A_1) The two masses $m_j (j = 1, 3)$ are stationary before impacting with the main mass m_2.

(A_2) The mass $m_j (j \in \{1, 3\})$ is separated with main mass m_2 once it comes back to the displacement boundaries after impacting with m_2.

Before and after impacts, the motion of the mass m_2 will change, thus the motions of the mass m_2 are divided into three types:

(i) When the mass m_2 does not move together with any of the masses m_1, m_3 and the velocity of the mass m_2 is not equal to the conveyor belt's, such motion is called slipping-nonstick motion or free motion. For this case, the equation of motion for the mass m_2 is given as

$$\ddot{x} + d_2\dot{x} + c_2 x = a_2 \cos \Omega t + b_2 - f_2 \text{sgn}(\dot{x} - V), \tag{2.2}$$