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Quantum modelling of the learning curve

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ABSTRACT

Learning curves are tools for assessing and designing strategy and policy. The stakes in decisions on strategy and policy may be high and the reliability of the curves therefore becomes a serious issue. The purpose of this paper is to show that the curve and the learning rates for an unperturbed system emerge from fundamental findings in quantum theory, non-equilibrium thermodynamics and cybernetics. One conclusion is that the learning curve phenomenon pervades all industrial activities performed by operationally closed systems. The learning system is modelled as a spinor representing two superposed operations, computing and producing. The phase difference between the operations is important for system behaviour.

1. Background and objective

Learning curves show how cumulative experience of producing, operating, and using a technology in a competitive environment consistently improves performance of the technology. The curves are widely used in industry (Jaber, 2010) for production management and strategic analysis and since the end of 1990s also for energy policy (IEA, 2000; Junginger et al., 2010). The price development of photovoltaic (PV) modules provides an illustrative example of the power of cumulative market experience, in this case supported by government deployment programmes (IEA, 2003). As cumulative sales of PV modules for solar electricity increased from 300 kW in 1976 to 2 GW in 2002 prices dropped from 64 to 3 USD(2001)/W_p (Schaeffer et al., 2004). However, achieving such price reductions requires large investments in learning and the reliability of learning curves in forecasting the effects of such investments has been seriously questioned (Nemet, 2009: Nordhaus, 2014). A thorough theoretical understanding of the curves would reduce such concerns. This paper addresses the theoretical basis for the learning curve.

Most theoretical analyses view the learning curve from the perspective of mainstream economics. Within this perspective, the curve is seen to express some useful correlations and learning involves, e.g., sampling of potentially cost-reducing processes representing technological, managerial, or behavioural alternatives to ongoing operations (for an overview, see Wene, 2016). This paper takes a cybernetic perspective and explores the hypothesis that the learning curve expresses fundamental properties of organizations in a competitive environment (Wene, 2015). The ultimate task is to ground the learning curve and its properties in established theories, namely quantum theory and non-equilibrium thermodynamics. Cybernetics provides the framework for this task as well as the entry points into quantum theory and non-equilibrium thermodynamics (Wene, 2007, 2008a, 2013).

Quantum modelling to explain macro-phenomena outside of physics and chemistry is today a strongly emerging field of research. Founders of quantum mechanics showed a genuine interest in such applications of quantum theory (Bohr, 1958; Schrödinger, 1944). Present advances in quantum experimentation, quantum computing, information theory and cryptography have spurred research into cognition, decision theory, logic, finance, medicine and social science (e.g., Dubois, 2002, 2017; Busemeyer et al., 2006; Khrennikov, 2006, 2010, 2017; Asano et al., 2011; Lambert-Mogiliansky & Dubois, 2015; Yamato et al., 2017). Examples of approaches are the fractaquantum model recursively applying quantum formalism on different scales (Dubois, 2002, 2017) and the Växjö model

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applying both classical and quantum contextual probabilistic models (Khrennikov, 2010). However, the work on the learning curve presented here builds on findings in cybernetics with roots in theoretical biology rather than quantum information theory (Varela, 1979, 1984; von Förster, 1984, 2003). The need to consider closure, circular causality and self-organising systems leads to quantum theory and operator formalism (von Förster, 1984; Wene, 2007, 2008a).

Learning as expressed by the learning curve pervades all industrial activities (BCG, 1968; Wene, 2016). We define and investigate one learning system (LS) that represents all organizations running such activities. Five concepts are used to characterise and analyse the learning system: *act-to-learn, non-trivial machine, eigentime, operational closure and entropy*. The two first concepts are directly linked to the observed shape of the curve and provide the starting point for the analysis, while the three last ones are the key concepts in the search for a theoretical foundation for the learning curve.

The learning curve emphasises that learning needs action. Using the output-input language of the machine metaphor the learning curve equation can be written as

$$P^{-1} = \text{input/output} = C_0 \cdot (\text{cumulative output})^{-E} \tag{1}$$

P is performance[1] and C_0 and E are constants. The experience parameter, E, is important, because it determines the improvement in performance from increasing cumulative output. The parameter E provides the *learning rate*, which is the relative improvement in performance for each doubling of cumulative output.[2]

Eq. (1) demonstrates that improving the capacity for effective action of the learning system requires production of output. Learning is not a function of calendar time but of implementation, i.e., interacting with the environment. In stressing *act-to-learn* the curves are consistent with major models for individual and organisational learning (Morgan, 1986; Kim, 1993; Espejo, Schuhmann, Schwaninger, & Bilello, 1996).

However, Eq. (1) also demonstrates that learning is a process with a history. Learning is a function of *cumulative* output, i.e., the output is continually fed back into the learning system changing its capacity for effective action. The learning system is a *non-trivial machine* (von Förster, 2003) with ability to control and change its internal operations to improve its performance. This implies that the learning system has *operational closure* (Maturana and Varela, 1980, 1992; Varela, 1979), i.e., the system has complete control of all its operations. All operations are connected in a closed network that does not accept any instructions from outside the network (Maturana & Varela,1980). The operations therefore form interconnected loops that are characterized by circular causality (von Förster, 2003, pp. 229–246) and the network is self-organizing (von Förster, 1984).

Operational closure is the key rationale both for the efforts of quantum modelling and for the need to consider non-equilibrium thermodynamics to explain the learning curve.

The operationally closed learning system mimics two defining features of quantum theory. Firstly, in the same way as the particle-wave dualism in quantum mechanical systems, operational closure severely reduces the degrees of freedom of the system. As a result, the system organizes itself into eigenstates, whose properties are observed or measured as eigenbehaviour or eigenvalues (von Förster, 1984; Varela, 1984).[3] Consistent with basic postulates in quantum mechanics (see e.g., Dicke & Wittke, 1960, pp. 90–103) we can describe the learning system by a state function and associate any observable with an operator, whose eigenvalues provide the outcomes of measurements. Secondly, the circular causality creates a "*circulus creativus*" (von Förster, 2003, p. 230) that braids all operations into a coherent system. In the quantum formalism, the state of the operationally closed learning system is therefore a superposition of operations. The distinction between pure and mixed states becomes meaningful because a measurement will turn a pure learning state into a mixed state (cf., Prigogine, 1980, pp. 60–61; Lambert-Mogiliansky & Dubois, 2015). The view of LS as a quantum system is consistent with the fractaquantum hypothesis (Dubois, 2002, 2017).

Wene (2007, 2008a) considered the operationally closed learning system to derive the spectrum of learning rates for the unperturbed case. Based on the Eq. (1) an operator for the experience parameter, E, was designed. The state of the learning system was represented by two orthogonal vectors in the complex plane, corresponding to operations related to performance and cumulative output, respectively. The spectrum for E was obtained as the limiting values for recursive operations on the state function. The result reproduces the measured learning rate of technologies for which the learning system is operationally closed and active in an environment with equilibrium markets, such as for PV-modules up to 2002.[4] The derived eigenvalues also reproduce the most probable values for learning rates in distributions of measured learning rates. However, the dispersion of learning rates around the eigenvalues is large indicating perturbations from the environment in the form of, e.g., changing regulations, external research and market instabilities. The system adapts to these perturbations through the mechanism of double closure (von Förster, 2003, pp. 211–246). Assuming linear interference between the two operations and Poisson distribution of perturbations, Wene (2010, 2011) obtains a good fit to the measured distributions. However, the approach provides no information on the effect on individual technologies from specific perturbations.

In the thermodynamic perspective the self-organizing learning system represents a paradox. As von Förster (2003, pp. 1–19) points out in a seminal paper from 1959: "There are no such things as self-organizing systems!". Seen by an outside observer, the self-organizing system appears to spontaneously reduce its entropy in contradiction to the Second Law of Thermodynamics. Any solution

[1] Performance is defined as P = output/input. However, the learning curve literature uses the inverse P^{-1} = input/output, which can, e.g., be measured in USD/kW or workdays/installed unit.

[2] The learning rate, LR, is obtained from $LR = 1–2^{-E}$ and is the parameter generally used to characterize learning in the literature.

[3] The cybernetic literature expresses the self-organisation of operationally closed system as a closure rule: "Every operationally closed system has eigenbehaviours" (Varela, 1984).

[4] A "silicon bubble" appeared after 2002 when the price of silicon ingots for PV-modules increased by a factor of ten, see e.g. Wene (2015).