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Free vibration analysis of viscoelastic nanotubes under longitudinal magnetic field based on nonlocal strain gradient Timoshenko beam model

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ABSTRACT

In this paper, the free vibration of viscoelastic nanotube under longitudinal magnetic field is investigated. The governing equation is formulated by utilizing Timoshenko beam model and Kelvin-Voigt model based on the nonlocal strain gradient theory. The local adaptive differential quadrature method (LADQM) is applied in the analyzing procedure. We also investigated the influences of the nonlocal parameter, structural damping coefficient, material length scale parameter and the longitudinal magnetic field on the natural frequencies of the system. The results of this research may be helpful for understanding the potential applications of nanotubes in Nano-Electromechanical System.

1. Introduction

Carbon nanotubes (CNTs) are essential structural elements used in several emerging Nano-Electromechanical System(NEMS) applications [1,2], like nano-oscillators [3], nano-scale clocks [4], parametric amplifiers [5], nano resonators [6] and so on.

As the size of CNTs is extremely small, the material microstructure's nanoscale size effect becomes important. In recent years, by considering the effect of material length, several theories have been proposed to investigate the behavior of nanostures, like nonlocal elasticity theory [7–23], strain gradient theory [24,25] and modified couple stress theory [26,27]. However, recent researches show that the individual nonlocal elastic theory or strain gradient elasticity theory has some limitations on identifying the size-dependent effect of CNTs [28–30]. The nonlocal elastic models can account for softening stiffness with increasing nanoscale parameter. However, the stiffness enhancement effect, which can be observed from both the experimental observation and the gradient elasticity (or modified couple stress) theories, cannot be characterized explicitly. Assuming that the materials cannot be modeled as collections of points, the gradient elasticity theories provide the classical equations of elasticity with additional higher-order strain gradient terms. The nonlocal elasticity theory and the gradient theory are quite different in describing physical characteristics of materials and structures at nanoscale. To overcome these limitations, Lim et al. [30] proposed the nonlocal strain gradient theory, which combined the nonlocal elastic theory with the strain gradient theory. It was shown that the results based on this theory is highly consistent with molecular dynamics simulation (MDS) results and has numerous applications in nanostructures. Much worked has been done based on the nonlocal strain gradient theory, either for wave phenomenon and vibration properties of nanoparticles, nanoscale beams, nanoshells or nanoplates [31–36]. Ebrahimi et al. [33] researched the wave propagation of an inhomogeneous functionally graded nanoplate subjected to nonlinear thermal loading by means of nonlocal strain gradient theory. Free vibration of nonlocal strain gradient beams which is made of functionally graded material was analyzed by Li et al. [34]. Based on the nonlocal strain gradient theory, Zhen and Zhou [35] investigated the transverse wave propagation in fluid-conveying viscoelastic single-walled carbon nanotubes (SWCNTs) with surface effect under multi-physics fields. The above literature show that the nonlocal strain gradient theory is reliable in describing CNTs' size effect. Karami et al. [36] investigated the wave dispersion in anisotropic doubly-curved nanoshells based on the nonlocal strain gradient theory and a high-order shell theory.

Experiments showed that CNTs exhibit viscoelastic properties when temperature ranges from $-196\,^{\circ}$C to $1000\,^{\circ}$C [37]. Chang and Lee [38] studied vibration behavior of a simply supported carbon nanotube using the nonlocal viscoelasticity theory in consideration of the thermal and foundation effects. Pang et al. [39] investigated the transverse wave propagation of viscoelastic SWCNTs adhered by surface materials based on nonlocal elasticity and Kelvin-Voigt model. Tang et al. [40] researched the viscoelastic wave propagation in an embedded viscoelastic single-walled carbon nanotubes(SWCNT) utilizing the nonlocal

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strain gradient theory.

Furthermore, the actual working environment of nanotubes often contains complex external physical factors, such as magnetic field, electric filed, and temperature field. In some applications of nanoengineering, the study on dynamic characteristic of CNTs under magnetic field is useful. The effect of longitudinal magnetic field on wave dispersion characteristics of equivalent continuum structure of SWCNTs embedded in elastic medium is studied by Narendar et al. [41]. Hosseini and Goughari [42] investigated the effect of a longitudinal magnetic field on the transverse vibration of a magnetically sensitive SWCNT conveying fluid. Based on nonlocal strain gradient theory and Rayleigh beam theory, the wave propagation analysis in viscoelastic SWCNTs is carried out by Li et al. [43], and the influences of longitudinal magnetic fields and surface effects on the properties of wave propagation are discussed. Karličić et al. [44] studied the nonlinear vibration and dynamic stability of a simply supported SWCNT under the influence of both the longitudinal magnetic field and time-varying axial load within the framework of the nonlocal elasticity theory.

To the best of the authors' knowledge, for the viscoelastic nanotubes under longitudinal magnetic field, the vibration analysis based on nonlocal strain gradient Timoshenko beam model has not been addressed. In this paper, we consider a cantilevered Timoshenko model to analyze the vibration characteristics of viscoelastic nanotubes under longitudinal magnetic field based on nonlocal strain gradient theory. The local adaptive differential quadrature method (LADQM) is used in analysis. The influences of the nonlocal parameter, structural damping coefficient, material length scale parameter and the longitudinal magnetic field on the natural frequencies are all elucidated. The results of this research may be helpful for understanding the potential applications of CNTs in NEMS.

2. Nonlocal strain gradient model

According to the nonlocal strain gradient theory [30], the total stress fields are composed of two parts, i.e. the classical nonlocal stress $\boldsymbol{\sigma}$ and the higher-order nonlocal stress $\boldsymbol{\sigma}^{(1)}$,

$$\mathbf{t} = \sigma - \nabla\boldsymbol{\sigma}^{(1)}. \tag{1}$$

The classical nonlocal stress $\boldsymbol{\sigma}$ and the higher-order nonlocal stress $\boldsymbol{\sigma}^{(1)}$ are defined as

$$\boldsymbol{\sigma} = \int_V \alpha_0(\mathbf{x}', \mathbf{x}, e_0 a)\mathbf{C}\colon \boldsymbol{\varepsilon}(\mathbf{x}')dV, \tag{2}$$

$$\boldsymbol{\sigma}^{(1)} = l^2\int_V \alpha_1(\mathbf{x}', \mathbf{x}, e_1 a)\mathbf{C}\colon \nabla\boldsymbol{\varepsilon}(\mathbf{x}')dV, \tag{3}$$

where $\alpha_0(\mathbf{x}', \mathbf{x}, e_0 a)$ and $\alpha_1(\mathbf{x}', \mathbf{x}, e_1 a)$ are the nonlocal attenuation functions associated with the strain tensor $\boldsymbol{\varepsilon}$ and the strain gradient tensor $\nabla\boldsymbol{\varepsilon}$, respectively. ∇ is the Laplace operator, and $\mathbf{C}$ is the fourth-order elasticity tensor. e_0 and e_1 are constants related to the materials.a is the length of carbon-carbon bond for CNTs and $e_0 a$, $e_1 a$are nonlocal parameters. l is the material length scale parameter introduced to indicate the significance of higher-order strain gradient stress field.

Since the integral constitutive is difficult to solve, here a simplified differential form is used. Suppose that the two nonlocal kernel functions $\alpha_0(\mathbf{x}', \mathbf{x}, e_0 a)$ and $\alpha_1(\mathbf{x}', \mathbf{x}, e_1 a)$ satisfy the conditions given by Eringen [8], similarly, a more general and extended constitutive equation in a differential form can be used in the nonlocal functions

$$L_i = 1 - (e_i a)^2\nabla^2, \quad i = 0,1. \tag{4}$$

Suppose $e = e_0 = e_1$, by applying Eq. (4) to Eq. (1), we get

$$[1 - (ea)^2\nabla^2]\mathbf{t} = C\colon \varepsilon - l^2\nabla\mathbf{C}\colon \nabla\varepsilon. \tag{5}$$

For a beam type structure, according to literature [30,45], the constitutive relation can be simplified as:

$$[1 - (ea)^2\nabla^2]t_{xx} = (1 - l^2\nabla^2)E(z)\varepsilon_{xx}, \tag{6}$$

$$[1 - (ea)^2\nabla^2]t_{xz} = (1 - l^2\nabla^2)G(z)\gamma_{xz}. \tag{7}$$

Here $E(z)$ is the elastic modulus and $G(z)$ is the shear modulus. t_{xx} and t_{xz} denote the axial stress and the shear stress, respectively. ε_{xx} and γ_{xz} denote the axial strain and the shear strain respectively.

Literature have shown that the constitutive relation (6) and (7) can explain the size-dependent phenomena of nano-materials reasonably and agree well with the results by the molecular dynamics simulations [30].

3. Mathematical model of viscoelastic nanotube under longitudinal magnetic field

In this article, the vibration characteristics of viscoelastic nanotube is researched utilizing the nonlocal strain gradient Timoshenko beam theory. For Timoshenko beam, the axial strain is

$$\varepsilon_{xx} = z\frac{\partial\psi}{\partial x}, \tag{8}$$

and the shear strain is

$$\gamma_{xz} = \psi + \frac{\partial w}{\partial x}, \tag{9}$$

where z is the normal to the x-axis, ψ is the rotation angle of the beam cross section, and w is the transverse displacement of the beam.

Substitute Eqs (8) and (9) into Eqs (6) and (7), and consider the viscoelastic property of nanotube with Kelvin-Voigt model, then we get the following nonlocal constitutive relation

$$[1 - (ea)^2\nabla^2]t_{xx} = (1 - l^2\nabla^2)\left(1 + \tau_d\frac{\partial}{\partial t}\right)Ez\frac{\partial\psi}{\partial x}, \tag{10}$$

$$[1 - (ea)^2\nabla^2]t_{xz} = (1 - l^2\nabla^2)\left(1 + \tau_d\frac{\partial}{\partial t}\right)G\left(\psi + \frac{\partial w}{\partial x}\right), \tag{11}$$

where τ_d is the structural damping coefficient which represents the viscoelasticity of CNTs.

The nonlocal bending moment M and the nonlocal shear force Q are defined as

$$M = \int_A z t_{xx} dA, \tag{12}$$

$$Q = \int_A t_{xz} dA, \tag{13}$$

where A is the cross section of the tube.

Integrating Eqs. (10) and (11), we get

$$M - (ea)^2\frac{\partial^2 M}{\partial x^2} = EI\left(1 - l^2\frac{\partial^2}{\partial x^2}\right)\left(1 + \tau_d\frac{\partial}{\partial t}\right)\frac{\partial\psi}{\partial x}, \tag{14}$$

$$Q - (ea)^2\frac{\partial^2 Q}{\partial x^2} = \kappa GA\left(1 - l^2\frac{\partial^2}{\partial x^2}\right)\left(1 + \tau_d\frac{\partial}{\partial t}\right)\left(\psi + \frac{\partial w}{\partial x}\right), \tag{15}$$

where κ is the shear correction factor which depends on the material.

The general equations of nonlocal viscoelastic Timoshenko beam, which include the external Lorentz force of longitudinal magnetic field are

$$\rho A\frac{\partial^2 w}{\partial t^2} = \frac{\partial Q}{\partial x} + q, \tag{16}$$

$$\rho I\frac{\partial^2\psi}{\partial t^2} = \frac{\partial M}{\partial x} - Q, \tag{17}$$

where ρ is the mass density, t is time, and I is the second moment of the cross section. q represents the external Lorentz force induced by longitudinal magnetic field [44] and it is expressed as

$$q(x, t) = \eta A H_x^2\frac{\partial^2 w}{\partial x^2}, \tag{18}$$

where η denotes the magnetic permeability, and H_x is the component of