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Cluster-daughter overlap as a new probe of alpha-cluster formation in medium-mass and heavy even–even nuclei

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A B S T R A C T

We study the possibility to use the cluster-daughter overlap as a new probe of alpha-cluster formation in medium-mass and heavy even–even nuclei. We introduce a dimensionless parameter $\mathfrak{D}$, which is the ratio between the root-mean-square (rms) intercluster separation and the sum of rms point radii of the daughter nucleus and the alpha particle, to measure the degree of the cluster-daughter overlap quantitatively. By using this parameter, a large (small) cluster-daughter overlap between the alpha cluster and daughter nucleus corresponds to a small (large) $\mathfrak{D}$ value. The alpha-cluster formation is shown explicitly, in the framework of the quartetting wave function approach, to be suppressed when the $\mathfrak{D}$ parameter is small, and be favored when the $\mathfrak{D}$ parameter is large. We then use this $\mathfrak{D}$ parameter to explore systematically the landscape of alpha-cluster formation probabilities in medium-mass and heavy even–even nuclei, with $\mathfrak{D}$ being calculated from experimentally measured charge radii. The trends of alpha-cluster formation probabilities are found to be generally consistent with previous studies. The effects of various shell closures on the alpha-cluster formation are identified, along with some hints on a possible subshell structure at $N = 106$ along the Hg and Pb isotopic chains. The study here could be a useful complement to the traditional route to probe alpha-cluster formation in medium-mass and heavy even–even nuclei using alpha-decay data. Especially, it would be helpful in the cases where the target nucleus is stable against alpha decay or alpha-decay data are currently not available.

1. Introduction

Alpha-cluster formation plays a key role in understanding structures and reactions of light, medium-mass, and heavy/superheavy nuclei across the nuclide chart [1–6]. For light nuclei, the existence of alpha-cluster structures was suggested in the late 1930s [7,8]. This idea is then explored in detail by generations of nuclear physicists, and inspires many important theoretical developments, such as resonating group method [9], generator coordinate method [10], antisymmetrized molecular dynamics [12], orthogonality condition method [11], THSR (Tohsaki–Horiuchi–Schuck–Röpke) wave function [13], etc. It is found that there could be alpha-cluster structures in ground states of light nuclei such as $^{8}\text{Be} = \alpha + \alpha$, $^{20}\text{Ne} = \alpha + {}^{16}\text{O}$, $^{44}\text{Ti} = \alpha + {}^{40}\text{Ca}$ [4], as well as in the famous Hoyle and Hoyle-like excited states of self-conjugate nuclei near alpha-particle disintegration thresholds [13–17]. The study of alpha-cluster formation in heavy/superheavy nuclei could date back to Rutherford's discovery of alpha decay more than one hundred years ago [18]. Till today, alpha decay is still an important direction being continuously developed [6,19–23], from which we gain lots of information on alpha-cluster formation in heavy/superheavy nuclei [20,24–37]. Especially, it is found that alpha-cluster formation probabilities change significantly at magic numbers $N = 126$ and $Z = 82$. Alpha-cluster formation could also exist in the medium-mass nuclei. Possible candidates include $^{46,54}\text{Cr} = \alpha + {}^{42,50}\text{Ti}$ [38,39], $^{90}\text{Sr} = \alpha + {}^{86}\text{Kr}$ [40], $^{92}\text{Zr} = \alpha + {}^{88}\text{Sr}$ [40], $^{94}\text{Mo} = \alpha + {}^{90}\text{Zr}$ [40–43], $^{96}\text{Ru} = \alpha + {}^{92}\text{Mo}$ [40], $^{98}\text{Pd} = \alpha + {}^{94}\text{Ru}$ [40], $^{136}\text{Te} = \alpha + {}^{132}\text{Sn}$ [44], etc. Recently, there are also systematic studies on the landscape of alpha-cluster formation probabilities in medium-mass nuclei using alpha-decay data [45].

Unlike alpha clustering in light nuclei which could be studied by using microscopic methods, probing alpha clustering in medium-mass and heavy/superheavy nuclei is more challenging thanks to intrinsic difficulties in solving quantum systems with a large number of nucleons. In literature, alpha-cluster formation in medium-mass and heavy/superheavy nuclei is usually investigated systematically by exploiting experimental data on alpha de-

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cay. However, neither all the nuclei with alpha-cluster structures permit alpha decays, nor all the nuclei permitting alpha decays have their decay widths be measured. Therefore, new probes of alpha-cluster formation besides those based on the alpha decay are necessary. In this note, we would like to study the possibility to adopt the cluster-daughter overlap as a new probe of alpha-cluster formation in medium-mass and heavy nuclei, motivated by the observation that alpha-cluster formation is suppressed when the cluster-daughter overlap is large and favored when the cluster-daughter overlap is small. In Section 2, we analyze the relation between the cluster-daughter overlap and the alpha-cluster formation quantitatively within the framework of the quartetting wave function approach [46–48], which is a microscopic method proposed recently to estimate alpha-cluster formation probabilities. The inclusion of shell-model properties in the quartetting wave function approach is also discussed recently in Ref. [49]. We introduce a dimensionless parameter $\mathfrak{D}$ to measure quantitatively the degree of the cluster-daughter overlap. In Section 3, the $\mathfrak{D}$ parameter is then adopted to explore the landscape of alpha-cluster formation probabilities in medium-mass and heavy even–even nuclei, with $\mathfrak{D}$ being calculated directly from experimentally measured charge radii. Special attentions are paid to the closed-(sub)shell effects. In Section 4, we give the conclusions.

2. Cluster-daughter overlap versus alpha-cluster formation in the light of quartetting wave function approach

We first study the relation between the cluster-daughter overlap and the alpha-cluster formation probability within the framework of the quartetting wave function approach. The quartetting wave function approach is inspired by the THSR wave function for light self-conjugate nuclei, and has been applied successfully in studying alpha-cluster formation in heavy/superheavy nuclei. Within this framework, we divide the wave function of the four valence nucleons uniquely as the product of the center-of-mass (COM) part and the intrinsic part. Following Refs. [46–49], by adopting the local-density approximation and ignoring the derivative terms of the intrinsic wave function, the Schrödinger equation for the COM wave function could be given by

$$-\frac{\hbar^2}{2M_\alpha}\nabla^2_{\mathbf{r}}\Phi(\mathbf{r}) + W(\mathbf{r})\Phi(\mathbf{r}) = E\Phi(\mathbf{r}). \tag{1}$$

$W(\mathbf{r})$ is the effective potential felt by the COM motion of the alpha cluster. A detailed derivation of Eq. (1) could be found in Ref. [46]. The alpha-cluster formation probability P_α could be then obtained as follows:

$$P_\alpha = \int \mathrm{d}^3\mathbf{r}\,|\Phi(\mathbf{r})|^2\,\Theta[\rho_B^{\mathrm{Mott}} - \rho_B(\mathbf{r})]. \tag{2}$$

Here, $\rho_B^{\mathrm{Mott}} = 0.02917\ \mathrm{fm}^{-3}$ is the Mott density higher than which the alpha cluster is believed to dissolve and merge with the daughter nucleus. The root-mean-square (rms) intercluster separation is given by

$$R_i \equiv \langle R^2\rangle_i^{1/2} = \left[\int_0^\infty \mathrm{d}r\, r^2\, |\phi(r)|^2\right]^{1/2}, \tag{3}$$

where $\phi(r)$ is the normalized radial wave function of $\Phi(\mathbf{r})$. To measure the degree of the cluster-daughter overlap quantitatively, we introduce a dimensionless parameter $\mathfrak{D}$ to measure the relative size between the rms intercluster separation R_i and the sum of the rms point radii of the alpha particle $R_\alpha \equiv \langle R^2\rangle_\alpha^{1/2}$ and the daughter nucleus $R_d \equiv \langle R^2\rangle_d^{1/2}$ (see Fig. 1 for a schematic representation),

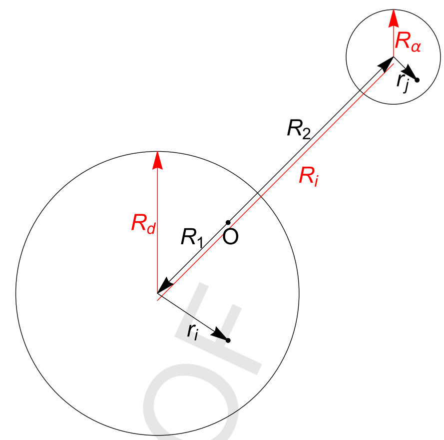

Fig. 1. A schematic representation of the cluster-daughter system. In this work, we assume that the centers of mass of the alpha particle and the daughter nucleus coincide with their centers of charge. R_α and R_d are the rms point radii of the alpha particle and the daughter nucleus. R_i is the rms intercluster separation. O is the center of mass of the parent nucleus. $\mathbf{R}_1$ ($\mathbf{R}_2$) is the displacement vector between O and the center of mass of the daughter nucleus (alpha particle). $\mathbf{r}_i$ ($\mathbf{r}_j$) is the displacement vector between the center of mass of the daughter nucleus (alpha particle) to a representative nucleon inside the daughter nucleus (alpha particle).

$$\mathfrak{D} = \frac{R_i}{R_\alpha + R_d}. \tag{4}$$

As a result, a large (small) overlap between the alpha cluster and the daughter nucleus corresponds to a small (large) value of the $\mathfrak{D}$ parameter.

In the rest part of this section, we would like to adopt the quartetting wave function approach to study the relation between the $\mathfrak{D}$ parameter and the alpha-cluster formation probability. We take $^{212}\mathrm{Po} = {}^{208}\mathrm{Pb} + \alpha$ as an example, which could be viewed as an alpha cluster moving on the top of the doubly magic nucleus $^{208}\mathrm{Pb}$ and has been investigated intensively by previous studies [41,42,50–53]. For the density distributions for the daughter nucleus, we adopt [47,54]

$$\rho_\pi(r) = \frac{0.0628948}{1+\exp[(r-6.68\ \mathrm{fm})/0.447\ \mathrm{fm}]}\ \mathrm{fm}^{-3}, \tag{5}$$

$$\rho_\nu(r) = \frac{0.0937763}{1+\exp[(r-6.70\ \mathrm{fm})/0.550\ \mathrm{fm}]}\ \mathrm{fm}^{-3}. \tag{6}$$

as the proton and neutron distributions, respectively. The critical radius that marks the dissolution of the alpha cluster is then given by $r_{\mathrm{cluster}} = 7.43825$ fm. In other words, the alpha cluster persists only for $r > r_{\mathrm{cluster}}$ and dissolves once $r < r_{\mathrm{cluster}}$. When $r > r_{\mathrm{cluster}}$, the effective potential $W(\mathbf{r}) = W^{\mathrm{ext}}(\mathbf{r}) + W^{\mathrm{intr}}(\mathbf{r})$, with $W^{\mathrm{ext}}(\mathbf{r})$ being the external potential inside which the alpha cluster moves and $W^{\mathrm{intr}}(\mathbf{r})$ being the intrinsic energy of the alpha cluster inside the nuclear medium. The external potential $W^{\mathrm{ext}}(\mathbf{r})$ could be approximated by $W^{\mathrm{ext}}(\mathbf{r}) = V_{\mathrm{M3Y}}(\mathbf{r}) + V_{\mathrm{C}}(\mathbf{r})$ following the double-folding procedure. The microscopic M3Y nucleon–nucleon interaction takes the form $V_{\mathrm{NN}}(s) = c\exp(-4s)/(4s) - d\exp(-2.5s)/(2.5s)$. For the alpha emitter $^{212}\mathrm{Po}$, the two free parameters c and d could be determined by fitting the emission energy Q_α and the alpha-decay half-life $T_{1/2}$. The intrinsic potential $W^{\mathrm{intr}}(\mathbf{r})$, on the other hand, is given by $W^{\mathrm{intr}}(\mathbf{r}) = W^{\mathrm{Pauli}}(\mathbf{r}) + E_\alpha^{(0)}$, with $W^{\mathrm{Pauli}}(\mathbf{r}) = 4515.9\rho_B(\mathbf{r}) - 100935\rho_B^2(\mathbf{r}) + 1202538\rho_B^3(\mathbf{r})$ [46] and $E_\alpha^{(0)} = -28.3$ MeV being the energy of the alpha particle in the vacuum. When $r < r_{\mathrm{cluster}}$, the cluster state of the four valence nucleons merges with the shell-model state of the daughter nucleus. In the local density approximation, we have $W(\mathbf{r}) = \mu_4$, with μ_4

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