Annals of Physics 398 (2018) 1–8

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Classical defocussing of world lines in higher dimensions

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A R T I C L E I N F O

Article history:
Received 20 February 2018
Accepted 26 August 2018
Available online 10 September 2018

Keywords:
World lines
Defocussing
Kaluza–Klein

A B S T R A C T

A five-dimensional gravity theory, motivated by the brane-world picture, with Kaluza scalar in the 5-dimensional metric as $g_{55}(r)$; $r = \sqrt{x^2 + y^2 + z^2}$, is considered near the possible singularity (small distance scales where gravity is strong) and is shown to give rise to a positive contribution to the Raychaudhuri equation. This inhibits the focusing of world lines and contributes to non-focusing of the worldlines in the 5-dimensional space. It is also shown that the results extend to time dependent cases such as those relevant for black hole interiors and cosmology.

1. Introduction

An interesting question in the field of gravitation is whether there is a simple way to avoid the singularities present in certain classical solutions. The Standard Cosmological Model based on Einstein's theory of gravitation, for example, implies that the universe began with a big bang singularity. The fascinating arena of black holes also possess singularities. Within the framework of Einstein's theory, the above singularities cannot be avoided without imposing exotic matter of some sort. This can be understood, for instance, by the Raychaudhuri equation [1] which in the absence of torsion, exhibits focusing of geodesics converging to the singularities [2]. In order to avoid such singularities without imposing exotic matter, one has to go beyond Einstein's theory of gravitation. This has been

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https://doi.org/10.1016/j.aop.2018.08.017
0003-4916/

investigated in the brane-world scenario and the recent studies indicate possible avoidance of the big bang singularity [3–5].

Studies in string theory motivated non-singular cosmologies [6,7] can avoid the big bang singularity. It has also been shown that in effective loop quantum gravity theories singularities can be avoided [8,9] . Ellis and Maartens [10], and Ellis, Murugan and Tsagas [11] proposed the emergent scenario in which the universe stays in a static past eternally and then evolves to a subsequent inflationary era, suggesting that the universe originates from Einstein static state rather than a big bang singularity. An emergent scenario has been made possible in the modified theories of gravity such as $f(R)$ gravity, loop quantum gravity [12] and in Einstein–Cartan theory [13]. *These studies motivate the consideration of higher dimensional gravity as a possible candidate for avoiding a singularity*. Further motivation is provided by the trace anomaly of Conformal Field Theory dual to a 5-dimensional Schwarzschild AdS geometry [14] in which H^4 (H being the Hubble parameter) terms are present in the equation for $\dot{H}$ and which leads to an infinite age of the universe, avoiding the singularity. Similar resolution of the singularity comes from the corrections to Raychaudhuri equation in the brane world scenario [15], in the approach using the 'generalized uncertainty principle' of quantum gravity [16] and in the quantum corrected Friedmann equations [17,18]. An interesting analysis from black hole–brane interactions has been done in [19].

While a theory of quantum gravity is far from being realized, a quantum corrected Raychaudhuri equation has been proposed by Das [20] and this was the basis to obtain the corrected Friedmann equation for $\dot{H}$ by Ali and Das [21] which avoids the big bang singularity, predicting infinite age for the universe. The said corrections to the Raychaudhuri equation cause defocussing of the geodesics thereby avoiding the singularity. We find in this manuscript that there is also a simple classical mechanism to produce a similar defocussing term in 5-dimensional world.

From the above studies, it is clear that in order to avoid the singularity, one needs to modify gravity such that defocussing of the geodesics occurs. One way to modify Einstein's theory of gravity is to consider five dimensional gravity (without electromagnetic fields) near the singularity (small scales where gravity is strong), a minimum modification.

A direct way to understand the possible avoidance of certain space-like singularities is to consider the Raychaudhuri equation. This equation in 4-dimensional gravity is $\frac{d\Theta}{ds} = -\frac{\Theta^2}{3} - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}u^{\mu}u^{\nu} + (\dot{u}^{\mu})_{;\mu}$, where $\Theta = u^{\mu}_{;\mu}$ characterizing the volume of the collection of particles with 4-velocity u^{μ} as they fall under gravity. That is, Θ provides a description of the expansion or contraction of a material body containing streamlines. The quantity $\sigma_{\mu\nu}$ is the symmetric tensor representing the shear, $\omega_{\mu\nu}$ is the antisymmetric tensor representing the vorticity and the last term $(\dot{u}^{\mu})_{;\mu}$ vanishes when the particles travel on their geodesics in 4-d theory. The vorticity causes expansion while the shear contraction. In the absence of vorticity or exotic matter, the geodesics contract or focus causing the universe to have a beginning a finite time ago, creating the big bang singularity or black hole spacelike singularity [2]. One could obtain solutions with shear and no vorticity; but not with vorticity and no shear [22] in Einstein's theory. The singularity theorems of Penrose and Hawking use this feature to state that there is an inevitable spacetime singularity [2,22]. In the absence of vorticity and shear, if the last term $(\dot{u}^{\mu})_{;\mu}$ exists and is positive, then defocussing of the world lines occurs thereby softening, or potentially avoiding the singularity.

Thus, attempts to avoid the singularity require either use of complicated field theoretic models of matter or modified gravity. The quantum corrections to the Raychaudhuri equation in [15,16,20] achieve this, preventing focusing of geodesics. This feature in cosmological considerations led to avoiding the big bang singularity with the universe without a beginning. Since certain black hole interiors have similar causal structure, the arguments may also apply to those singularities.

It is worthwhile to examine whether a non-focusing term of the world lines similar to the above studies could emerge *classically* in higher dimensional gravity with minimum modifications.

The aim of this paper is to show a defocussing term of world lines arises by considering five-dimensional Kaluza theory with fifth dimension at small scales (near the singularity) where gravity is expected to be strong—a minimal modification of the 4-d gravity. We hasten to add that we interpret our results modestly, in that we show that a defocussing term appears similar to what is found in the models cited above, but from the consideration of classical general relativity without exotic or