Nonlinear Analysis: Real World Applications 46 (2019) 12–28

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

Multi-dimensional bipolar hydrodynamic model of semiconductor with insulating boundary conditions and non-zero doping profile

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ARTICLE INFO

Article history:
Received 30 June 2018
Accepted 26 August 2018
Available online xxxx

Keywords:
Global existence
Asymptotic behavior
Bipolar hydrodynamic model
Insulating boundary conditions
Non-zero doping profile

ABSTRACT

In this paper, we are concerned with the global existence and asymptotic behavior of smooth solution to the multi-dimensional bipolar hydrodynamic model of semiconductor with insulating boundary conditions and non-zero doping profile. For any space dimension, we prove the global solutions exist and converge to the corresponding stationary with an exponential decay rate when the initial data are close to a certain steady state, which may not be constants. We also get a L^2 decay estimate which can be valid for any L^∞ weak entropy solution. Moreover, we give specific conditions on initial data and doping profile when space dimension $d = 1$.

1. Introduction

We are interested in the global existence and asymptotic behavior of smooth solutions to the following bipolar hydrodynamic model of semiconductor

$$\begin{cases} \partial_t n_1 + \nabla \cdot \mathbf{j}_1 = 0, \\ \partial_t \mathbf{j}_1 + \nabla \cdot (\dfrac{\mathbf{j}_1 \otimes \mathbf{j}_1}{n_1}) + \nabla p_1(n_1) = n_1 \mathbf{E} - \mathbf{j}_1, \\ \partial_t n_2 + \nabla \cdot \mathbf{j}_2 = 0, \\ \partial_t \mathbf{j}_2 + \nabla \cdot (\dfrac{\mathbf{j}_2 \otimes \mathbf{j}_2}{n_2}) + \nabla p_2(n_2) = -n_2 \mathbf{E} - \mathbf{j}_2, \\ \nabla \cdot \mathbf{E} = n_1 - n_2 - D(x), \end{cases} \tag{1.1}$$

where $(x,t) \in \Omega \times (0,\infty)$ with Ω being a bounded domain in R^d, $d \geq 1$. The unknowns $n_1(x,t)$, $\mathbf{j}_1(x,t)$ (respectively $n_2(x,t)$, $\mathbf{j}_2(x,t)$) represent the scaled partial density and current density of electrons (respectively holes). The unknown function $\mathbf{E}$ denotes the electric field, which is generated by the Coulomb force of particles. If we introduce the electrostatic potential ϕ, then $\mathbf{E} = \nabla\phi$. In this paper, we consider the

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https://doi.org/10.1016/j.nonrwa.2018.08.011
1468-1218/

isothermal case $p_i(n_i) = n_i (i = 1, 2)$, which is of importance in industry. The symbols $\bigotimes$ and $(\nabla\cdot)$ denote the Kronecker tensor product and the divergence in R^d. $D(x) \neq 0$ is the doping profile, which means the density of impurities in semiconductor materials. We suppose

$$D(x) \in C(\Omega), \quad D^* = \sup_x D(x) \geq \inf_x D(x) = D_*, \tag{1.2}$$

otherwise, we may assume $D(x)$ is approximated by a smooth function. In this paper, we consider problem (1.1) with the initial conditions

$$n_i(x, 0) = n_{i0}(x) > 0, \quad \mathbf{j}_i(x, 0) = \mathbf{j}_{i0}(x), \quad i = 1, 2, \tag{1.3}$$

and the following insulating boundary conditions

$$\mathbf{j}_i(x, t) \cdot \mathbf{n}\Big|_{\partial\Omega} = 0, \qquad \frac{\partial\phi}{\partial\mathbf{n}}\Big|_{\partial\Omega} = 0, \tag{1.4}$$

where $\mathbf{n}$ is the unit outward normal vector along $\partial\Omega$.

In the past thirty years, the mathematical study on hydrodynamic model of semiconductor has attracted a lot of attention. For the unipolar model, after Degond–Markowich considered the existence and uniqueness of the subsonic steady solution in [1], Gamba [2] investigated the stationary transonic solution. For the time dependent model, Hsiao–Yang [3], Luo–Natalini–Xin [4] and Guo–Strauss [5] proved the existence of global smooth solutions near a given steady state. However, Chen et al. [6] proved the existence of the local generalized solution and gave the blow up phenomenon of this solution. Therefore, it is necessary to study weak solutions, [7–13] considered the existence and large time behavior of weak solution to the unipolar hydrodynamic model of semiconductor, respectively. For more results about the unipolar model of semiconductor, we can refer to the review [14–25].

For the bipolar hydrodynamic model, the existence and asymptotic behavior of smooth solutions to the system (1.1) have been studied by [26–29] when the doping profile $D(x) \equiv 0$. In addition, Ju [30] solved the same problem with a bounded domain and insulating boundary condition. Furthermore, in the case of non-zero doping profile, we can refer Donatelli et al. [31] and Mei et al. [32]. However, none of above is studied on the system (1.1) with non-zero doping profile and insulating boundary conditions. The fundamental reason is that we have not got the existence of corresponding steady states. Recently, Tsuge [33] and Yu [34] obtained the existence of steady state to system (1.1) with Ohmic contact boundary and insulating boundary conditions, respectively. As for the existence of weak solutions and its relaxation behaviour we can see [35,36]. Moreover, Huang–Li [37], Yu–Zhan [38] and Li–Yu [39] give the large time behavior framework of weak solution. For more researches on correlation models we can refer to the references [37,40–42].

In this paper, our main goal is to prove, when the doping profile is not zero, the global existence and exponential convergence of multi-dimensional bipolar hydrodynamic model of semiconductor with insulating boundary conditions. That is, the global smooth solution of problem (1.1)–(1.4) exists and converges to the corresponding stationary system

$$\begin{cases} \nabla\tilde{n}_1 = \tilde{n}_1\tilde{\mathbf{E}}, \\ \nabla\tilde{n}_2 = -\tilde{n}_2\tilde{\mathbf{E}}, \\ \nabla\cdot\tilde{\mathbf{E}} = \tilde{n}_1 - \tilde{n}_2 - D(x), \\ \tilde{\mathbf{E}}\cdot\mathbf{n}\Big|_{\partial\Omega} = 0, \end{cases} \tag{1.5}$$

with an exponential decay rate.