



## Decision Support

## Role of specific energy in decomposition of time-invariant least-cost reservoir filling problem



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## ABSTRACT

A mathematical analysis highlighting the decomposition structure of the least-cost reservoir filling problem under time-invariant conditions is provided. It is shown, without loss of generality, that time invariance and unidimensionality of the state variable (for describing the evolution of the hydrodynamic system) are sufficient in order to achieve full (spatial and temporal) decomposition. Using this result, the role of specific energy in finding least-cost operational schedules for reservoir filling in a general “physically meaningful” hydrodynamic system is discussed.

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## 1. Introduction

Operational optimization of hydrodynamic systems has been, and continues to be, in the focus of extensive research. The area covers a broad range of applications ranging from drinking water processing and distribution (D'Ambrosio, Lodi, Wiese, & Bragalli, 2015; Ghaddar, Claeys, Mevissen, & Eck, 2017; Ghaddar, Naoum-Sawaya, Kishimoto, Taheri, & Eck, 2015; Naoum-Sawaya, Ghaddar, Arandia, & Eck, 2015) to wastewater treatment (Hou, Li, Xi, & Cen, 2015; Wei, He, & Kusiak, 2013), irrigation (Reca, Garca-Manzano, & Martinez, 2015) and energy production (Kusakana, 2016; Steffen & Weber, 2016). The field addresses the operational aspects of hydrodynamic systems where given the system topology, the task is to derive an operational policy for the active hydrodynamic components (pumps and valves) so that the related costs are minimal subject to operational constraints.

Among the operational optimization problems related to hydrodynamic systems, the optimal loading of the system's storage(s) has received much attention. This problem, referred to as *least-cost reservoir filling*, has high importance in pumped storage hydroelectricity (Rehman, Al-Hadhrani, & Alam, 2015; Steffen & Weber, 2016) but arises in water supply (Bene & Hős, 2012; Lindstedt & Karvinen, 2016; Sarbu, 2016) and other applications as well. The study (Ghaddar et al., 2015) on least-cost pump scheduling related to water supply recognized that the related optimization problem exhibits a special decomposition structure which can be exploited

by Lagrangian relaxation to provide an approximate solution. The optimization problem involved the direct minimization of the pumping energy cost subject to system constraints (water network behavior, reservoir dynamics etc.). A study presented in Bene and Hős (2012) approaches a similar but simplified problem, where pumping energy cost is minimized during the loading of a water reservoir. One of the main contributions of this paper is that the energy minimization is addressed by the (re)formulation of the objective function of the related mathematical programming problem. Namely, the energy cost is not directly minimized, instead it is incorporated in the optimization objective utilizing the concept of *specific energy*. By definition, specific energy is the energy required to convey a unit mass (volume) of fluid. Using this formulation, the authors observed that the problem can be decomposed into smaller subproblems which can be solved individually.

Although the two outlined approaches are fundamentally different, both achieve problem decomposition by the manipulation of the objective function. In Ghaddar et al. (2015), the problem decomposition relies on solid (mathematical) theory, while the specific energy approach is supported only by some simulations. In Bene and Hős (2012) and Bene (2013), it is demonstrated through an extensive parameter study that the utilization of specific energy implies decomposition structure for the least-cost reservoir filling problem under the assumption that the energy tariff and the water consumption do not vary in time.

Consequently, Bene and Hős (2012) and Bene (2013) intuitively suggest that assuming time-invariant consumption and energy tariff the specific energy approach implies decomposability of the least-cost reservoir filling problem. While this result is accepted by the community (Coelho & Andrade-Campos, 2014; Mala-Jetmarova,

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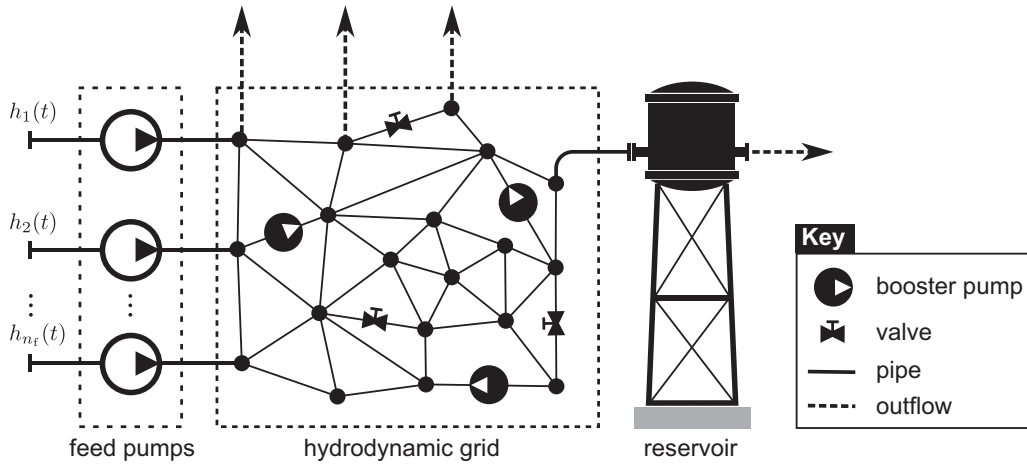


Fig. 1. Hydrodynamic system of interest.

Sultanova, & Savic, 2017), it seems that a gap has emerged between theory and practise. To the best of the authors' knowledge no study has been published which would clarify the relation between specific energy cost formulation, time-invariant conditions and the decomposition structure of the least-cost reservoir filling problem. On the other hand, the numerous attempts to solve the optimization problem indicate that much of the research is devoted to the problem formulation, approximation and solution derivation, while considerably less attention has been paid to the analysis of the underlying problem.

To fill this gap, this paper is devoted to the analysis of the least-cost reservoir filling problem class. A rigorous mathematical framework is developed in order to understand and reveal the decomposition structure of this problem class under time-invariant conditions. The main contributions are as follows: It is shown that under time-invariant conditions (a) the least-cost reservoir filling problem exhibits full (spatial and temporal) decomposition structure; (b) direct cost minimization and the minimization of specific energy are identical problems, different formulations of the same objective; and (c) problem decomposition structure is not a result of specific energy formulation, i.e., decomposition can be achieved without the manipulation of the objective function. Consequently, the results achieved so far in this field are extended, generalized and explained in detail thereby providing a deeper insight into the underlying problem class.

The paper is organized as follows: In Section 2 the mathematical programming problem of reservoir filling is introduced. Section 3 details the basic properties of the least-cost reservoir filling problem through the principle of optimality. In Section 4 the time-invariant problem is introduced and the decomposition structure is revealed by the relaxation of the principle of optimality, while Section 5 addresses the role of specific energy formulation in least-cost reservoir filling. Finally, Section 6 provides a summary of the results presented in this paper and draws the conclusions.

#### Notation

The following notation is adopted: the symbols  $\mathbb{R}$ ,  $\mathbb{R}^+$  and  $\mathbb{R}_{\geq 0}$  denote the set of real numbers, the set of positive reals and the set of non-negative reals, respectively. Similarly,  $\mathbb{Z}^+$  denotes the set of positive integers and  $\mathbb{Z}_N^+ := \{1, \dots, N\}$  a finite set of positive integers less than or equal to  $N \in \mathbb{Z}^+$ . Vectors  $\mathbf{x} \in \mathbb{R}^n$  are displayed as boldface lower case Latin letters and scalars  $x \in \mathbb{R}$  in normal italic typeset. The transpose of a vector  $\mathbf{x}$  is denoted by  $\mathbf{x}'$ . Given a func-

tion  $f: \mathcal{X} \rightarrow \mathcal{Y}$ , the set  $\mathcal{Y}$  is called the image of  $\mathcal{X}$  under  $f$ , where  $\mathcal{Y} := \{y \in \mathbb{R}^n \mid \exists x \in \mathcal{X} \text{ such that } y = f(x)\}$ . Given a set  $\mathcal{X}$ ,  $\dim_{\mathbb{R}}(\mathcal{X})$  denotes the dimension of  $\mathcal{X}$  over  $\mathbb{R}$ .

## 2. Problem formulation

In this section the least-cost reservoir filling problem is formulated.

The hydrodynamic system of interest (see Fig., 1) consists of three main components: (1) pumping stations (feed, booster), (2) hydrodynamic grid and (3) a reservoir (storage). Pumping stations are comprised by individual pumps providing the required pressure and flow conditions for the system. The hydrodynamic grid is a network of (active and passive) hydrodynamic components such as pipes, pumps and valves. The grid can be described by a directed graph, where edges represent hydrodynamic components indicating flow directions and vertices (nodes) are the joints of hydrodynamic components (Burgschweiger, Gnädig, & Steinbach, 2009). The reservoir can accumulate, release and store matter. It is considered as a stand-alone unit or may represent the aggregated (total) storage capacity of the system.

Let us assume that the hydrodynamic system of interest has  $n_p \in \mathbb{Z}^+$  individual pumps in total (including feed pumps as well as booster pumps) and a given topology (e.g., pumps organized in pumping stations are running in parallel, series or a combination of both). Similarly, the hydrodynamic grid has  $n_v \in \mathbb{Z}^+$  installed valves and  $n_c \in \mathbb{Z}^+$  nodes. For each node, a bounded flow  $0 \leq w_i(t) \leq w_i^{\max}(t)$  (kg/h) (referred to as *outflow*) is assigned defining the loss of matter due to (for example) consumption, leakage, drain or evaporation. The outflow vector is characterized by the individual outflows  $\mathbf{w}(t) := (w_1(t), \dots, w_{n_c}(t))' \in \mathcal{W}$  at time  $t \in \mathcal{T}$  (h), where  $\mathcal{T} := \{\tau \in \mathbb{R} \mid \tau \geq 0\}$ . Regarding outflow, the following assumption is formulated:

**Assumption 1.** The outflow trajectory  $\{\mathbf{w}(t) \mid t \in \mathcal{T}\}$  is known or can be forecasted with high accuracy.

Focusing on operational aspects, the system of interest has  $n_p + n_v$  manipulated inputs in total, including the vector of pump speeds/states  $(\omega_1, \dots, \omega_{n_p}) \in (\Omega_1, \dots, \Omega_{n_p})$  and the vector of valve opening degrees  $(\zeta_1, \dots, \zeta_{n_v}) \in (\Upsilon_1, \dots, \Upsilon_{n_v})$ . Depending on the element type, the variable domains  $\Omega_i$  and  $\Upsilon_j$  can be integer valued finite (i.e., on-off type pumps and valves) or real valued compact sets (i.e., variable speed pumps and continuous valves) including zero in their interior. Using these, the vector of manipulated inputs  $\mathbf{u}(t) := (\omega_1(t), \dots, \omega_{n_p}(t), \zeta_1(t), \dots, \zeta_{n_v}(t))' \in \mathcal{U}$  is

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