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Discrete and continuous models for static and modal analysis of out of plane loaded masonry

D. Baraldi *, A. Cecchi

Department of Architecture Construction Conservation DACC, Università IUAV di Venezia, Italy

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ABSTRACT

A critical review of analytical and numerical models for studying masonry out of plane behaviour is presented. One leaf historical masonry, composed by rigid blocks arranged regularly with dry or mortar joints, is considered. Discrete model with rigid blocks, Love-Kirchhoff and Reissner-Mindlin plate models and 3D heterogeneous FEM are adopted. An existing simple and effective discrete model is adopted and improved by applying matrix structural analysis techniques for static and modal analysis of masonry walls in the elastic field, but the formulation allows to account also for material nonlinearity. Elastic parameters of both plate models are based on an existing compatible identification between 3D discrete model and 2D plate models. Static and modal analysis of masonry walls with several boundary conditions are carried on, numerical tests account for in plane size of heterogeneity and structure thickness by means of in and out of plane scale factors. Results show that discrete model is simple and effective for representing masonry behaviour, especially when size of heterogeneity is smaller than overall panel size. Decreasing in plane scale factor, plate models converge to the discrete one, but the Reissner-Mindlin one shows a better convergence and also allows adopting a simple FE for performing numerical analysis.

1. Introduction

Masonry is a structural material obtained by composition of natural or artificial blocks connected by dry or mortar joints. For this type of material, size of heterogeneity (or size of block) is often not negligible with respect to the global size of a structural element; then, several ad-hoc models have been developed in the last decades adopting different approaches.

The model that may appear as the more simple one among others for representing masonry behaviour is a heterogeneous Finite Element (FE) model. The first example of such a model type was limited to the in plane case, characterised by blocks modelled with FE quadrilateral elements and joints modelled with one-dimensional elements [1,2]. Small improvements of these initial works were performed by Tzamtzis & co-workers by defining three-dimensional elements and joints, but limiting the field of analysis to the in plane case [3,4]. However, the limits of this approach are represented by the large number of degrees of freedom involved and the consequent computational effort for the analysis of macro-scale problems.

Another class of numerical models frequently adopted for representing masonry behaviour is discrete modelling, that is characterised by rigid or deformable elements in contact, in order to represent natural or artificial blocks in contact by means of dry or mortar joints. Into this class of models, the discrete element method (DEM) is one of the most representative approaches. Such a method is characterised by distinct elements, with finite size and independent degrees of freedom, that can be subject to finite displacements and rotations; moreover, contacts between elements can vary during analysis and are automatically recognized by the model. DEM had been introduced by the pioneering works of Cundall in the field of rock mechanics [5,6], that started considering the plane case and created the well-known computer code UDEC [7], and continued modelling three-dimensional problems [8,9], creating the computer code 3DEC [10]. Several developments in the DEM field are represented by the combination of discrete and finite elements [11,12] accounting for block deformability; however these models are still limited to in plane analysis. Another type of discrete models is represented by those adopted in Discrete Deformation Analysis (DDA, [13]) that are characterised by deformable blocks by means of uniform strain and stresses in plane state. This model was extended to the 3D case even if at preliminary stage [14,15].

The discrete models cited above were created for modelling granular materials and for studying rock mechanics. In several cases, such models were extended to the field of masonry structures with results comparable with those obtainable with other

* Corresponding author.
E-mail addresses: danielebaraldi@iuav.it (D. Baraldi), cecchi@iuav.it (A. Cecchi).

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classes of models; the work of Lemos [16] presents a deep review of DEM applied to masonry structures, furthermore the recent book edited by Bagi et al. [17] collects an up-to-date review of DEM for masonry and other discrete approaches.

However, historical masonry is frequently made of strong and rigid natural or artificial blocks and weak, thin and deformable mortar joints. For this reason, numerical models, characterised by rigid blocks, with deformability concentrated at mortar joints or dry contacts, subject to small displacements that do not vary contact topology, should be sufficiently accurate and effective. Effectiveness is given by the small number of degrees of freedom involved in the analysis that allows to model structures starting from small masonry panels to building facades and bridges. It is worth noting that these models cannot be defined DEMs but still remain discrete models. Considering the in plane case, this type of model was adopted by many authors in linear and non-linear fields [18–22]. In particular, the discrete and heterogeneous models introduced by Cecchi and Sab [20] were extended to the out of plane case by also performing homogenisation procedures [23–25]. Similarly, the 'rigid-body-spring-model' introduced by Casolo was effectively extended to the out of plane case, in linear and nonlinear fields [26,27] and it was also compared with a homogenised model [28].

Heterogeneous and discrete models are often linked with continuous materials equivalent to masonry, obtained by means of identification or homogenisation procedures. Continuous models are another class of models that are generally adopted for studying masonry behaviour at macroscale level, when both heterogeneous and discrete models start to be inapplicable due to the huge number of degrees of freedom involved in the analysis of masonry buildings. Considering the in plane case, standard Cauchy models were obtained applying periodic homogenisation techniques and considering the elastic behaviour of both brick and mortar [20,24,29]; moreover, micropolar or higher order continua were taken in consideration [30–35]. Considering the out of plane case, Stefanou et al. [36] performed a 3D Cosserat homogenisation of regular masonry composed by rigid elements in the linear elastic field and then proposed a FE formulation for Cosserat elastic plates [37]. However more generally, research in the 3D or out of plane field has been generally focused on nonlinear masonry behaviour for performing nonlinear, limit and stability analysis of walls, facades and buildings [27,38–46]. Recently, Ferreira et al. [47] presented an accurate literature review related to the analysis of unreinforced masonry out of plane loaded.

Considering the field of analysis based on homogenisation or identification procedures, plate models are often adopted for modelling out of plane masonry behaviour. In particular, the already cited works of Cecchi and Sab [23,24] show an identification procedure that is based on the balance of internal work in the discrete model and in the continuous one for a class of regular motions. In this field, an important problem is represented by how kinematic, dynamic and constitutive prescriptions of a discrete system are transferred to the continuous one. Hence, constitutive functions of the plate model may be different. For example, a Love-Kirchhoff plate model was proposed by Cecchi and Sab [20] for the case of both rigid and deformable blocks by means of homogenisation procedures, whereas Cecchi and Sab [24] studied both Love-Kirchhoff and Reissner-Mindlin plate models for rigid blocks connected by elastic interfaces by means of a 3D discrete model and homogenisation procedures.

Recently, Baraldi et al. [48] have presented a review of several numerical models, heterogeneous, discrete and continuous, that may be adopted for modelling the mechanical behaviour of masonry, with particular attention to out of plane loaded panels having a specific regular texture. The present work aims to extend the initial review by adding further information about out of plane displacement and rotation fields obtained with linear static analysis. Moreover, this work aims to extend the campaign of numerical tests to the field of modal analysis, by means of a simple and effective approach for studying the discrete system, based on the determination of the stiffness matrix of the masonry assemblage. In addition, analytic solutions relative to natural frequencies of homogenised plates simply supported along edges are presented.

Hence in this work, numerical evaluation of the differences between a discrete model with rigid blocks, heterogeneous FEM and homogenised plate models is carried on for several case studies, performing static and modal analysis and considering several boundary conditions. The effect of varying in plane heterogeneity size (block width respect to panel width) is considered, as it has been done for the in plane case by several authors [32,35,49]; moreover, the effect of block aspect ratio (block width with respect to block height) and out of plane scale factor (block thickness with respect to panel width) are taken into account.

In order to represent the behaviour of historical masonry, characterised by block stiffness larger than mortar stiffness and joint thickness smaller with respect to block size, the discrete model adopted in this work and the corresponding homogenised plate models are based on the following hypotheses: (i) masonry structure composed by infinitely rigid blocks subject to small displacements and with fixed contact topology, (ii) mortar joints modelled as elastic interfaces. It is worth noting that the elastic behaviour considered may be not correct for studying masonry structures, given that such structures present a strong nonlinear behaviour even at low stress levels; however, the proposed review represents an initial step for performing numerical tests of out of plane loaded panels, that can be extended to the nonlinear field in further developments of this work, adopting, for example, a Mohr-Coulomb yield criterion for interface actions, following the numerical tests recently dealt with by authors both for the in and out of plane cases [50,51]. Moreover, the proposed campaign of modal analyses will allow to perform structural identification of masonry specimens by comparing numerical results with laboratory or in situ tests.

2. Discrete model

This work considers a regular and periodic masonry assemblage, characterised by equal rigid blocks arranged regularly with aligned horizontal joints and vertical joints staggered by block half width. This model is defined as Discrete Rigid Block Model (DRBM). A representative elementary volume (REV) is considered (Fig. 1), characterised by a generic block $B_{i,j}$ surrounded by six blocks by means of six interfaces or joints S_{k_1,k_2}, with k_1, k_2 = ±1 for horizontal interfaces and k_1 = ±2, k_2 = 0, for vertical interfaces (Fig. 1). Block dimensions are: a (height), b (width) and s (thickness). It is worth noting that this contact topology is assumed to be fixed during the analysis. Considering rigid block and small displacements

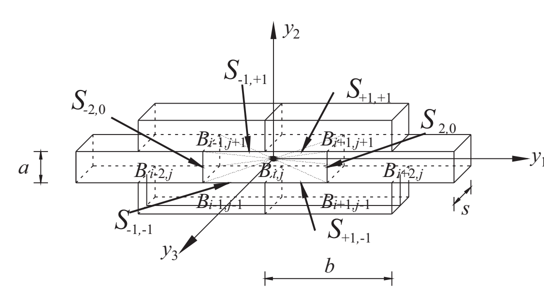

Fig. 1. Discrete rigid block model (DRBM), running bond Representative Elementary Volume (REV).