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Journal of the Franklin Institute 000 (2018) 1–35

www.elsevier.com/locate/jfranklin

Solutions to linear bimatrix equations with applications to pole assignment of complex-valued linear systems

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Received 8 February 2018; received in revised form 31 May 2018; accepted 31 July 2018
Available online xxx

Abstract

We study in this paper solutions to several kinds of linear bimatrix equations arising from pole assignment and stability analysis of complex-valued linear systems, which have several potential applications in control theory, particularly, can be used to model second-order linear systems in a very dense manner. These linear bimatrix equations include generalized Sylvester bimatrix equations, Sylvester bimatrix equations, Stein bimatrix equations, and Lyapunov bimatrix equations. Complete and explicit solutions are provided in terms of the bimatrices that are coefficients of the equations/systems. The obtained solutions are then used to solve the full state feedback pole assignment problem for complex-valued linear system. For a special case of complex-valued linear systems, the so-called antilinear system, the solutions are also used to solve the so-called anti-preserving (the closed-loop system is still an antilinear system) and normalization (the closed-loop system is a normal linear system) problems. Second-order linear systems, particularly, the spacecraft rendezvous control system, are used to demonstrate the obtained theoretical results.

1. Introduction

In this paper we continue to study complex-valued linear systems introduced in [41]. Complex-valued linear systems refer to linear systems whose state evolution dependents on both the state and its conjugate (see Section 2.1 for a detailed introduction). There are several

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https://doi.org/10.1016/j.jfranklin.2018.07.015
0016-0032/

reasons for study this class of linear systems [41], for example, they are naturally encountered in linear dynamical quantum systems theory [13,26], can be used to describe the so-called symmetric linear systems [4], and can be used to model any real-valued linear systems with lower dimensions (see Section 2.2 for a detailed development). Analysis and design of complex-valued linear systems have been studied in our early paper [41], where some fundamental problems such as state response, controllability, observability, stability, pole assignment, linear quadratic regulation, and state observer design, were solved. The conditions and/or methods obtained there are based on bimatrices associated with the complex-valued linear system, which is mathematically appealing.

The pole assignment problem for complex-valued linear system was solved in [41] by using coefficients of the so-called real-representation system, for which any pole assignment algorithms for normal linear systems can be applied. In this paper, we will continue to study the pole assignment problem for complex-valued linear systems by establishing a different method. Our new solution is based on solving the so-called (generalized) Sylvester bimatrix equation whose coefficients are bimatrices associated with the complex-valued linear system. Our study on linear bimatrix equations and their applications in pole assignment for complex-valued linear systems has been inspired by early work for normal linear systems. For example, the (normal) Sylvester matrix equation was utilized to solve the pole assignment for normal linear system in [3], and the generalized Sylvester matrix equation was used in [9–12,42] to solve the (parametric) pole assignment problem for normal linear systems, descriptor linear systems, high-order linear systems, and even periodic linear systems. Other topics regarding matrix equations and their applications to system theory can be found, for instance, in [5,8,16,21,32,33,38–40].

We will show that the pole assignment problem for a complex-valued linear system has a solution if and only if the associated (generalized) Sylvester bimatrix equation has a nonsingular solution. Thus the main task of this paper is to provide complete and explicit solutions to the homogeneous (generalized) Sylvester bimatrix equation. The solutions we provided have quite element expressions that use the original coefficient bimatrices and a right-coprime factorization (in the bimatrix framework) of the system. We also provide solutions to non-homogeneous Sylvester bimatrix equations and Stein bimatrix equations which include the Lyapunov bimatrix equation as a special case.

We are particularly interested in pole assignment for the so-called antilinear system studied recently in [34–37]. By our approach we first provide closed-form solutions to the associated (generalized) Sylvester bimatrix equations, and then consider two different problems, namely, the anti-preserving problem which ensures that the closed-loop system is still (or equivalent to) an antilinear system, and the normalization problem which guarantees that the closed-loop system is (equivalent to) a normal linear system. The anti-preserving problem was firstly studied in [36]. However, we can provide complete solutions that use full state feedback rather than only normal state feedback used in [36]. We discovered that the anti-preserving problem is meaningful only for discrete-time antilinear systems (as studied in [36]) since any continuous-time antilinear system cannot be asymptotically stable. However, the normalization problem is valid for both continuous-time and discrete-time antilinear systems, and seems more interesting as the closed-loop system is (equivalent to) a normal linear system that is more easy to handle.

The remainder of this paper is organized as follows. The motivation and some preliminary results are presented in Section 2, where we will introduce the complex-valued linear system and the associated linear bimatrix equations. Complete and explicit solutions to the

Please cite this article as: B. Zhou, Solutions to linear bimatrix equations with applications to pole assignment of complex-valued linear systems, Journal of the Franklin Institute (2018), https://doi.org/10.1016/j.jfranklin.2018.07.015