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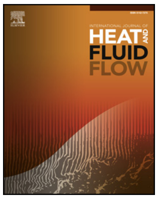

Topology optimization of heat sinks in a square differentially heated cavity

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ABSTRACT

Innovative designs of heat sinks are generated in the present paper through numerical optimization, by applying a material distribution topology optimization approach. The potential of the method is demonstrated in a two-dimensional differentially heated cavity, in which the heat transfer is increased by means of introducing a solid structure that acts as a heat sink. We simulate the heat transfer in the whole system by performing direct numerical simulations of the conjugated problem, i.e. temperature diffusion and convection in the entire domain and momentum conservation in the fluid surrounding the solid. The flow is driven by the buoyancy force, under the Boussinesq approximation, and we describe the presence of solid material as the action of a Brinkman friction force in the Navier–Stokes equations. To obtain a design with a given length scale, we apply regularization techniques by filtering the material distribution. Two different types of filters are applied and compared for obtaining the most realistic solution. Given the large scale of the problem, the optimization is solved with a gradient based method that relies on adjoint sensitivity analysis. The results show the applicability of the method by presenting innovative geometries that are increasing the heat flux. Moreover, the effect of various factors is studied: We investigate the impact of boundary conditions, initial designs, and Rayleigh number. Complex tree-like structures are favored when a horizontal temperature gradient is imposed on the boundary and when we limit the amount of solid volume in the cavity. The choice of the initial design affects the final topology of the generated solid structures, but not their performance for the studied cases. Additionally, when the Rayleigh number increases, the topology of the heat exchanger is able to substantially enhance the convection contribution to the heat transfer.

1. Introduction

In the electronics industry, one of the current engineering challenges is to enhance the thermal performances of natural convection cooled heat sinks (NCC HS). A typical configuration of this type of products is shown in Fig. 1 (i.e., a vertical NCC HS with parallel fins is cooling the electronics located on the back side). However, due to the miniaturization of electronic components and the increase of the thermal power density, new designs are sought. The reliability and durability of the natural convection cooling systems is very attractive for many industrial applications, mainly because of the low maintainability and operational costs. Therefore, as summarized in the following, the research in the optimization of these systems has rapidly evolved and is flourishing.

The first forms of the use of optimization techniques for the systematic analysis of NCC HS date back to the 1990s, when very simple optimization of the spacing between vertical plates have been amdeveloped using parametrization techniques (Bar-Cohen and Rohsenow, 1984). In the following years, increasingly complex geometrical configurations have been considered. For example, Morrison focused on simultaneously optimizing many sizing parameters of the heat sink (Morrison, 1992), and Bahadur on parameterizing more complex geometries like pin fins (Bahadur and Bar-Cohen, 2005). These optimization approaches are very powerful for fine-tuning purposes. For example, at the last stages of the product design development, when the topology is well defined and the optimal values of some free parameters are sought. In this context, the limited amount of degrees of freedom is advantageous and guarantees all outcomes of the optimization to have an equivalent variation of the topology of the device. The challenge is however to identify an innovative topology leading to a significant enhancement of the thermal performance. Due to the limited amount of degrees of freedom, parametric studies and boundary shape optimization are not suitable for the task: The strong dependency on the initial layout can hamper the identification of a cutting-edge design. Therefore, in the last decade, more advanced optimization techniques have become more attractive. On one side, the constructal method

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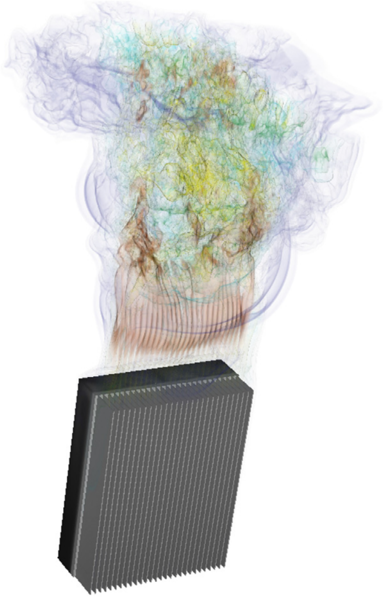

Fig. 1. Direct numerical simulation of the coupled conjugate heat transfer problem arising around a typical heat sink having vertical fins. The volume rendering of the velocity field, colored with the temperature scale, shows how the flow is rising to the top of the solid structure. The flow is driven by only natural convection: A heat source located behind the heat sink causes a temperature increase in the solid. The surrounding air is thus warmed up and flows upwards because of buoyancy. Despite the laminar flow through the fins, due to the low local Rayleigh number, the plume transitions to a turbulent regime above the heat sink.

proposed by Bejan gained attention (Bejan and Lorente, 2008), and, in contrast, the material distribution topology optimization method pioneered by Bendsøe and Kikuchi (1988). For the second method, the central idea is to compute the optimal spatial distribution of a composite material function. Every computational point becomes a degree of freedom, and the topology can be described without the need of “shape” functions (Jameson and Jameson, 2007). This numerical design optimization approach is very powerful and offers many advantages. It is flexible, implementable in already existing numerical software and allows for changes in the connectedness of the solid structure. This method was developed for the optimization of load-carrying elastic structures. It has been later exploited in a large spectrum of different applications, as presented by Bendsøe and Sigmund (2011) and Maute (2014).

For enhancing the performance of heat sinks, originally, topology optimization has been applied to simple conduction problems (Bendsøe and Sigmund, 2011). Later works started to take into consideration the effect of the surrounding fluid. At first, the contribution of the convection was accounted for just through a boundary condition, relying on empirical thermal heat transfer coefficients (Bruns and Tortorelli, 2001). However, it is not easy to justify the use of these average and effective coefficients in topology optimization. They are empirically obtained for certain configurations, specific conditions and type of solid-fluid interaction. The results obtained with topology optimization methods are adaptively modifying the interface between the two materials. Moreover, they may lead to an unexpected design, for which it is very unlikely to find the proper heat transfer coefficient. Therefore, when applying topology optimization to thermal fluid dynamic problems, it is beneficial to simulate both the solid and the fluid material to realistically capture the physics involved. The first attempts to describe the entire conjugate heat transfer were limited to Stokes problems (Borrvall and Petersson, 2003; Thellner, 2005; Koga et al., 2013). However, in the last years, Alexandersen et al. (2014) have proven the feasibility of applying this method to a complete conjugate heat transfer problem, in which the momentum conservation is governed by the Navier–Stokes equations. The control of the solid topology is achieved by introducing in the momentum equations a Brinkman friction term, which penalizes the velocity inside the structure (Brinkman, 1949). The solid can be seen as an ideal porous medium, whose material properties are mapped according to the material distribution function (Bruns, 2007). An alternative to this direct immersed boundary method (Goldstein et al., 1993) is the adoption of a level set method (Yaji et al., 2015). This approach allows for a sharp boundary definition, however, it has the disadvantage of requiring re-meshing (Zhou and Li, 2008) or a special boundary treatment (Villanueva and Maute, 2017).

In the present paper, which represents the first step towards the optimization of NCC HS, we consider a differentially heated cavity (DHC) in two dimensions, in which a swirling convective flow is induced by buoyancy (Le Quéré and Behnia, 1998; Xin and Le Quéré, 2001). We apply the material distribution topology optimization method by relying on mathematical programming techniques combined with direct numerical simulations (DNS). The natural convection flow arising from the conjugate heat transfer inside the cavity is computed with a high-order accurate numerical scheme, in the form of a solver based on the spectral element method: `Nek5000` (Fischer et al., 2008). In particular, we are interested in steady-state solutions. These are obtained by time marching with increased convergence rate (Citro et al., 2017). The design is represented by a binary scalar material distribution function ρ that can represent both the fluid ($\rho = 1$) and solid ($\rho = 0$) at each computational point in the domain under investigation. However, this model does not guarantee a mesh independent solution. Therefore, we apply standard restriction techniques to relax the problem and promote mesh independence. In this paper, we extend the work by Alexandersen et al. (2014) by first of all exploring the effect of more advanced filters as compared to the classical linear one (Bruns and Tortorelli, 2001). We thus implement a nonlinear fW-mean filter, as suggested by Hägg and Wadbro (2017). These two filter operations are used to calculate a smooth material distribution that is also defined for intermediate values $\tilde{\rho} \in [0, 1]$.

The objective of the optimization is to maximize the heat flux through the right side of the cavity; similar as expected in a real HS application. We solve the above mentioned problem by using a gradient based optimization algorithm (continuous adjoint sensitivity analysis approach).

Another difference with respect to the cases studied by Alexandersen et al. (2014) is the range of Rayleigh numbers Ra under investigation. In the three–dimensional simulation of the real NCC HS shown in Fig. 1 we have evaluated that the range of meaningful Ra has a lower bound of 6500 inside the heat sink, where the flow is laminar (Ra evaluated with respect to the hydraulic diameter in the pitch between fins). The upper limit corresponds to $72 \cdot 10^6$ for the flow exiting the heat sink (Ra evaluated with respect to the height of the heat sink). As in their case, we also limit ourselves to study the steady (time independent) flow inside the cavity. We therefore ignore the turbulent plume region developing above the NCC HS, and focus on the optimization of a flow in similar conditions to the one between the fins ($10^4 \leq Ra \leq 10^6$). For a DHC, this range of Ra has been extensively studied in the literature and the flow is hydro-dynamically stable (*i.e.*, there is a stationary solution to the governing equations) (Xin and Le Quéré, 1995).

The paper is organized as follows. The governing equations and the non-dimensionalization are presented in Section 2, the numerical methods used to solve the physical problem are described in Section 3, and the setup follows in Section 4. Section 5 presents the optimization algorithm and in Section 6 we discuss the results. In particular, we focus on the impact of the optimization domain, boundary condition, filter, initial designs, the Rayleigh number, and on the heat flux that the optimal geometries deliver.

At the end of the paper, Appendix A, Appendix B, and Appendix C