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Research article

# Robust output feedback distributed model predictive control of networked systems with communication delays in the presence of disturbance

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ABSTRACT

In this work, an output feedback cooperative distributed model predictive control is developed for a class of networked systems composed of interacting subsystems interconnected through their states, in which it handles bounded disturbances and time varying communication delays. A distributed buffer based prediction strategy is used to compensate bounded delays and predict those states, which are coupled between subsystems that their actual values may not available due to delays. In the design of robust distributed model predictive control, distributed moving horizon estimation is employed so that convergence and boundedness of the estimation error are ensured. Furthermore, robust exponential stability of the closed loop system is established. The effectiveness of the proposed method is illustrated using two interconnected continuous stirred tank reactors.

## 1. Introduction

Distributed model predictive control (DMPC) has been widely utilized for the control of complex networked systems such as power networks, traffic networks and chemical processes [1–6]. These networked systems composed of several interconnected subsystems which can communicate information through a transmission network. In DMPC, each subsystem has its own local controller but despite of decentralized MPC, interactions between subsystems are regarded. DMPC has the advantages of low computational burden than centralized MPC and higher performance than decentralized MP [2,3]. According to the cost function used by each subsystem, DMPC can be categorized into cooperative DMPC and non-cooperative DMPC. Cooperative DMPC, in which a local controller optimizes a global cost function, can account for plantwide stabilizing even for strongly interacting subsystems. Compared to the cooperative DMPC, the optimization problem in a non-cooperative DMPC is solved locally in each subsystem and nominal stability can be shown only for weekly interacting subsystems [7,8]. In addition to handling constraints, scales and interactions in multivariable processes, MPC can inherently advocate the case of transmission fail due to the packet losses. In such situation not only the first element but also a larger portion of the predicted control sequence is transmitted in the network [9].

In DMPC, each local controller inherently requires full knowledge of its states, but in practice, they may not be measurable or even may be corrupted with noise. Therefore, combining DMPC with an estimation framework namely output feedback DMPC is a practical alternative to state feedback DMPC. On the other hand, distributed estimation as a promising approach between decentralized and centralized estimation has been received a great deal of research interests [10,11]. In constrained DMPC, state estimation errors can lead to infeasibility; however, the nominal DMPC is feasible. Among different estimation methods, Moving Horizon Estimation (MHE) can handle system constraints and improve the estimation error bounds [10–14]. In essence, MHE is an online optimization problem based on the most recent data, therefore it has a high degree of robustness in the presence of uncertainties [12]. Moreover, MHE has been successfully applied in practical processes [15–17].

In the literatures, relatively few researches investigate DMPC subject to communication disruptions such as delays and packet losses [6,18–24]. State feedback DMPC in a non-ideal network connection has been discussed in Refs. [18–23]. Since the results of these researches are based on the availability of the state measurements of the entire system, they may not be applicable in practice. Combining DMPC with an estimation framework and considering communication disruptions, it enlarges the applicability scope of DMPC in the modern industries. Although, the mentioned problem is important, very few articles have worked in this field. To the best of the authors' knowledge, only [6,12] have addressed output feedback DMPC under communication disruptions. Output feedback DMPC of uncertain systems with randomly occurring actuator saturation and packet loss has been investigated in Ref. [24], where the distributed controller is obtained by defining the

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estimation error function and forming an augmented system to handle the dynamic output feedback control. In Ref. [6], state estimation based DMPC of large-scale systems with communication delay has been discussed in which a distributed Luenberger type observer has been designed for each subsystem to estimate the local states of each subsystem without considering state and output disturbances. Distributed MHE (DMHE) of nonlinear systems subject to communication delay and data loss has been studied in Ref. [11]. A framework for simultaneous constrained MHE and explicit MPC has been presented in Ref. [25]. To the best of the authors' knowledge, no result exists on the robust output feedback DMPC via MHE and considering communication disruptions. Motivated by the above observations, this paper focuses on the robust output feedback cooperative DMPC of linear networked systems coupled via states subject to time varying communication delays and bounded state and output disturbances. Each subsystem controller sends its designed predicted control sequence to the other subsystems through a delayed communication network. Using a buffer based approach and designing the distributed predictors, those states that are coupled between subsystems that their actual values are not available due to delays can be obtained. Moreover, the DMHE with pre-estimator is designed to estimate the local states of each subsystem. Using the predicted states and estimated states, the cooperative DMPC is designed by solving an optimization problem. The main contributions of the paper can be summarized as follows:

1. The distributed prediction strategy is proposed to compensate the network communication delays.
2. The constrained DMHE with pre-estimator is designed using the predicted states that are given by the prediction strategy. Moreover, an analytic dynamic expression for the estimation error is presented.
3. The robust output feedback cooperative DMPC is proposed based on the predicted and estimated states. In addition, the robust exponential stability of the closed loop networked system is analyzed.

The rest of this paper is organized as follows. In Section 2, the system model and the preliminary assumptions are presented. Also, a buffer based approach for delay compensation in the communication network is explained. Robust output feedback cooperative DMPC over communication network is designed in Section 3. In Section 4, dynamic equations and bounding sets of the estimation error for each local moving horizon estimator is derived. Then the robust exponential stability of the closed loop system is performed. The theoretical results are verified in Section 5. Finally, Section 6 concludes the paper.

**Notation.** Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the $n$ dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. $\|.\|$ refers to the Euclidean norm for vectors and induced 2-norm for matrices. Also, $\|.\|_S$ represents the $s$-weighted norm with respect to the matrix $S$. The superscripts "T" and "-1" indicate matrix transpose operation and inverse of any square matrix, respectively. The symbol $\mathbb{I}_M$ denotes the set of integers $1,\ 2,\ \ldots, M$. For matrix or vector $A_i$, $col(A_i)$ for $i\in\mathbb{I}_M$, denotes the matrix or vector $[A_1^T,\ \ldots, A_M^T]^T$. $blkdiag(A_i)$ for $A_i$, $i\in\mathbb{I}_M$ stands for a block-diagonal matrix with the matrices $A_i$, $i\in\mathbb{I}_M$ on the main diagonal and zeros elsewhere. $A>0$ means $A$ is a symmetric positive definite matrix. The symbol $\oplus$ denotes the Minkowski sum. Given a sequence set $\mathbb{X}^i$, $\prod_{i\in\mathbb{I}_M}\mathbb{X}^i \triangleq \mathbb{X}^1\times\ldots\times\mathbb{X}^M$. The symbol $r\mathbb{B}$ denotes a closed ball of radius $r>0$ created at origin. int($\mathbb{X}$) represents the interior of set $\mathbb{X}$. For a state vector $x$, $x^+$ is the successor state. A polyhedron is the (convex) intersection of a finite number of open and/or closed half spaces and a polytope is the closed and bounded polyhedron. A set $\mathbb{X}\subseteq\mathbb{R}^n$ is a $\mathbb{C}$-set if it is a compact and convex set that contains the origin in its interior. The set $\mathbb{X}\subseteq\mathbb{R}^n$ is Robust Positively Invariant (RPI) [26] for $x^+ = Ax + w;\ w\in\mathbb{W}\subseteq\mathbb{R}^n$ if $Ax + w\subseteq\mathbb{X}$ for all $w\in\mathbb{W}$ and all $x\in\mathbb{X}$.

## 2. Preliminaries

### 2.1. System description

Consider a linear networked system composed of $M$ subsystems coupled via states. Subsystem $i$ can receive information from its neighboring subsystems whose indices are denoted by $\mathbb{N}_i$ with $\mathbb{N}_i\in\mathbb{I}_M$ and $\mathbb{N}_i\neq\varnothing$. The nominal dynamic model (i.e. without considering disturbances and communication delays) of the $i$-th subsystem is given by the following discrete-time equations:

$$\bar{x}_{k+1}^i = A^{ii}\bar{x}_k^i + B^iu_k^i + \sum_{j\in\mathbb{N}_i} A^{ij}\bar{x}_k^j \tag{1a}$$

$$\bar{y}_k^i = C^i\bar{x}_k^i \tag{1b}$$

where $\bar{x}_k^i\in\mathbb{X}^i\subseteq\mathbb{R}^{n_x^i}$ is the state vector, $u_k^i\in\mathbb{U}^i\subseteq\mathbb{R}^{n_u^i}$ is the control input and $\bar{y}^i\in\mathbb{R}^{n_y^i}$ is the output vector. $A^{ii}$, $B^i$, $A^{ij}$ and $C^i$ are known constant matrices with appropriate dimensions. $\mathbb{X}^i$ and $\mathbb{U}^i$ are polyhedral and polytopic constraint sets, respectively, and both contain the origin as an interior point. It is assumed that $\mathbb{X}^i$ is a set of all states for which there exists a feasible control input lies in $\mathbb{U}^i$. For $i\in\mathbb{I}_M$, the pairs $(A^{ii},\ B^i)$ and $(A^{ii},\ C^i)$ are assumed to be stabilizable and detectable, respectively. The entire nominal system can be written as

$$\bar{x}(k+1) = A\bar{x}(k) + Bu(k), \tag{2a}$$

$$\bar{y}(k) = C\bar{x}(k) \tag{2b}$$

with state $\bar{x} = col(\bar{x}^i)\in\mathbb{X}\subseteq\mathbb{R}^{n_x}$, control input $u = col(u^i)\in\mathbb{U}\subseteq\mathbb{R}^{n_u}$ and measured output $\bar{y} = col(\bar{y}^i)\in\mathbb{R}^{n_y}$. $\mathbb{X} = \prod_{i\in\mathbb{I}_M}\mathbb{X}^i$ and $\mathbb{U} = \prod_{i\in\mathbb{I}_M}\mathbb{U}^i$ are state and input constraint sets, respectively. The matrix $A$ is defined as: $A\triangleq\begin{bmatrix} A^{11} & A^{12} & \cdots & A^{1M}\\ \vdots & \vdots & \ddots & \vdots\\ A^{i1} & A^{i2} & \ldots & A^{iM}\\ \vdots & \vdots & \ddots & \vdots\\ A^{M1} & A^{M2} & \ldots & A^{MM}\end{bmatrix}$. Also, $B\triangleq blkdiag(B^i)$ and $C\triangleq blkdiag(C^i)$. Note that if there is no interaction between subsystem $i$ and $j$, then the corresponding element in matrix $A$ is considered as zero.

### 2.2. Network description

The subsystems are assumed to exchange information via a shared communication network subject to time-varying delays. In the proposed method, each subsystem MPC controller has a control horizon of length $Nc$ and packages its predictive control sequence into one packet with a time-label and then sends it to the other subsystems over the network. The time delay between subsystem $i$ and subsystem $j$ is denoted by $\tau_k^{ij}$ at time step $k$. It is assumed that the time delays are integer multiples of sampling period and $d_{min}\le\tau_k^{ij}\le d_{max}$ where $d_{min}$ and $d_{max}$ are known positive integers representing lower bound and upper bound on the delays, respectively.

In this work, each local controller contains $(M-1)$ buffers that each of them appropriates to a subsystem and stores the corresponding packet until the arrival of the next packet. When a packet arrives, its time label is compared with the time label of the existing packet in the buffer. If a valid control packet arrived (i.e. the newly arrived packet is more recent than the existing packet), then the buffer updates, otherwise, the newly arrived packet will be discarded and containing of the buffer is shifted one to the left and then a zero is added to the right. This buffering mechanism is equal to a parallel-in serial-out shift register. For example, suppose that at time step $(k-1)$, the following valid predictive control sequences packet $\boldsymbol{u}_{k-1}^j = \{u_{k-1}^j,\ u_k^j,\ \ldots, u_{k+Nc-2}^j\}$ is written on the corresponding buffer of the $i$-th local controller. If at time step $k$, a non-valid packet with time delay $\tau_k^{ij}$ is arrived then the buffer is