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Research article

Robust finite-time consensus control for multi-agent systems with disturbances and unknown velocities

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ABSTRACT

In this paper, we investigated the finite-time consensus tracking problem for multi-agent systems with external bounded disturbances and input bounded disturbances and unknown velocities. Based on the Lyapunov finite-time theorem, a novel finite-time consensus control is constructed by using the backstepping method. For unknown velocities, the high-gain observer is used to estimate the velocity information. It is proved that the consensus can be achieved in finite time. The consensus shows fast response and strong robustness to various disturbances. Finally, the effectiveness of the results is illustrated by numerical simulations.

1. Introduction

Recently, the consensus of problem of multi-agent systems has attracted increasing attention due to its applications in multi-vehicles formation, sensory networks, distributed computation, and so on [1–3]. The consensus means that all agents reach an agreement on a state under a designed protocol based only on local relative information between neighboring agents [4]. Currently, the consensus problem can be roughly categorized into two classes, namely leader-follower consensus (consensus with a leader) [5–7] and leaderless consensus (consensus without leader) [8,9]. There have been many consensus algorithms [5,10–15] developed by synthesizing algebraic graph theory and control theory. An important topic in the study of the consensus problem is the convergence rate. Furthermore, the abovementioned literature mainly focuses on the asymptotical convergence rate. The literature shows the best asymptotical convergence is exponential with infinite settling time, i.e., the states cannot reach a consensus in finite time. In practical applications, it may be more desirable to achieve consensus tracking in finite time. Therefore, it is very useful to investigate the finite-time consensus tracking control for multi-agent systems. Compared with asymptotic consensus, the finite-time consensus provides not only a faster convergence rate but also stronger robustness to uncertainty and disturbance rejection [16–20].

The finite-time consensus control has been studied in a number of recently published papers, see for instance [4,9,21–27]. For example, Hui et al. addressed some necessary and sufficient conditions for finite-time semi-stability of homogeneous multi-agent systems [21]. Zhao et al. proposed a robust finite-time stability control for robotic manipulators by using backstepping method, which is proved by the finite-time Lyapunov stability theorem [28]. Li Shihua et al. designed a continuous distributed control algorithms for multi-agent systems described by double integrators based on the finite-time control technique [4]. Khoo et al. proposed a robust finite-time consensus tracking algorithm for multi-robot systems with input disturbances based on the terminal sliding mode control [22]. Similarly, Zhao et al. constructed a continuous consensus tracking control using a nonsingular terminal sliding mode scheme [23]. Liu et al. discussed the finite-time consensus problem for a class of time-varying nonlinear multi-agent systems, and proposed a finite-time controller based on the Lyapunov stability theorem [26]. He et al. constructed a finite-time consensus protocol by using the Lyapunov stability theorem [9]. Note that the abovementioned algorithms require velocity measurements to be available.

In practice, the velocity is difficult to obtain or cannot be precisely measured [29], which makes it difficult to achieve consensus in a finite time using only the relative position information. There are some results about finite-time consensus algorithms without velocities [7,29–32]. For example, Zhang et al. proposed a finite time observer-based controllers for multi-agent systems to achieve finite-time consensus with unavailable velocities [32]. With the existence of disturbances, Zhao and Duan designed a finite-time containment protocol that uses only relative position measurements [31]. Hua et al. investigated a finite-time consensus control for second-order multi-agent systems without velocity measurements [7]. However, methods presented in those papers cannot consider input disturbances. Motivated by the abovementioned results, this paper discusses the finite-time tracking problem of second-order multi-agent systems with external

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bounded disturbances, input bounded disturbances and unknown velocities by using the backstepping method.

The main contributions of this paper are as follows. First, a finite-time consensus for multi-agent systems is designed based on the finite-time Lyapunov stability theorem and the backstepping method motivated by paper [28,33]. Second, the external bounded disturbances and input bounded disturbances are considered in multi-agent systems. Finally, The high-gain observers are used to obtain velocity information, and the saturation input is introduced to eliminate the peaking phenomenon and make it more easily use in practice. In contrast to the previous works related to finite-time consensus, the proposed consensus control can enhance the robustness of multi-agent systems to various disturbances.

The rest of the paper is organized as follows. Section 2 gives some preliminaries on graph theory. The main results are discussed in Section 3. In Section 4, two numerical examples are given to illustrate the theoretical results. Section 5 gives the conclusions.

2. Preliminaries and model description

2.1. Graph theory notations

For a multi-agent system consisting of one leader and n followers. Let $G=\{\nu,\kappa\}$ be a directed graph, where $\nu=\{0,1,2,\cdots,n\}$is the set of nodes, node i represents the ith agent, $\kappa$is the set of edges, and an edge in G is denoted by an ordered pair $(i,j)\cdot(i,j)\in\kappa$ if and only if the ith agent can send information to the jth agent directly, but not necessarily vice versa. In contrast to a directed graph, the pairs of nodes in an undirected graph are unordered, where the edge (i,j) denotes that agent i and j can obtain information from each other. Therefore, an undirected graph can be viewed as a special case of a directed graph. A directed tree is a directed graph, where every node has exactly one parent except for the root, and the root has a directed path to every other node. A directed spanning tree of G is a directed tree that contains all nodes of G.

The matrix $A=(a_{ij})\in R^{(n+1)\times(n+1)}$where $a_{ij}>0$, if $(j,i)\in\kappa$and $a_{ij}=0$ otherwise, is called the weighted adjacency matrix of G with nonnegative elements. Let $D=\operatorname{diag}\{d_0,d_1,\cdots,d_n\}\in R^{(n+1)\times(n+1)}$be a diagonal matrix, where $d_i=\sum_{j=0}^{n}a_{ij}$for $i=0,1,\ldots,$ n. Then, the Laplacian of the weighted graph can be defined as

$$L=D-A\in R^{(n+1)\times(n+1)} \tag{1}$$

The connection weight between the ith agent and the leader is denoted by b_i with $b_i>0$ if there is an edge between the ith agent and the leader.

2.2. Finite-time stability theory

The Lyapunov finite-time stability theorem is discussed in Refs. [34,35].

Lemma 1. Consider the non-Lipschitz continuous nonlinear system$\dot{x}=f(x)$with $f(0)=0$. Suppose there are C^1 positive definite function $V(x)$ defined on a neighborhood of the origin and real numbers $c>0$, and $0<\alpha<1$, such that

(1) $V(x)$is positive definite;

$$\dot{V}(x)\le -cV^{\alpha}(x),\text{ where }\dot{V}(x)=\frac{\partial V}{\partial x}f(x). \tag{2a}$$

Then, the origin is a finite-time stable equilibrium, and the settling time, which depends on the initial state $x(t_0)=x_0$, satisfies

$$T(x_0)\le\frac{V^{1-\alpha}(x_0)}{c(1-\alpha)} \tag{2b}$$

for all x_0 in some open neighborhood of the origin.

Lemma 2. [33] Consider the nonlinear system $\dot{x}=f(x,u)$. Suppose that there exist continuous function $V(x)$, scalars $c>0$, $0<\alpha<1$ and $0<\varepsilon<\infty$ such that

$$V(x)\le -cV^{\alpha}(x)+\varepsilon \tag{3}$$

Then, the trajectory of system$\dot{x}=f(x,u)$ is practical finite-time stable.

2.3. Description of the second order multi-agent systems

In this paper, we consider a multi-agent system with leader and followers. The leader is active, and its behavior is independent of the followers. The dynamics of the leader are described as follows:

$$\begin{aligned}&\dot{x}_0=v_0,\quad x_0\in R^m\\&\dot{v}_0=u_0,\quad v_0\in R^m\end{aligned} \tag{4}$$

where x_0 is the position and v_0 is the velocity of the leader. The dynamics of the ith follower agent are described by

$$\begin{aligned}&\dot{x}_i=v_i,\qquad x_i\in R^m\\&\dot{v}_i=u_i+\delta_i,\quad v_i\in R^m,\quad i=1,\cdots,n\end{aligned} \tag{5}$$

where $u_i(i=1,\cdots,n)$represents the control inputs and δ_i represents the various disturbance, which is bounded, i.e., $\|\delta\|_\infty\le\bar{\bar{\delta}}$, $\bar{\bar{\delta}}>0$.

Definition 1. The multi-agent system is said to achieve second-order finite-time consensus if for any initial conditions

$$\lim_{t\to T}\|x_i(t)-x_0(t)\|=0,\ \lim_{t\to T}\|v_i(t)-v_0(t)\|=0$$

and

$$x_i(t)=x_0(t),\quad v_i(t)=v_0(t),\quad\forall\ t\ge T,\quad i=1,2,\cdots,n.$$

where T is a positive constant.

The system consists of $n+1$ agents, where an agent indexed by 0 acts as the leader and the other agents indexed by 1, …, n, are referred to as the followers. The topological relationships between the leader and the followers are described by a directed graph $G=\{\nu,\kappa\}$, with $\nu=\{0,1,\cdots,n\}$and the adjacent matrix

$$A=\begin{bmatrix}0&0&\cdots&0\\a_{10}&a_{11}&\cdots&a_{1n}\\\vdots&\vdots&\ddots&\vdots\\a_{n0}&a_{n1}&\cdots&a_{nn}\end{bmatrix}\in R^{(n+1)\times(n+1)} \tag{6}$$

Denote $\bar{G}=\{\bar{\nu},\bar{\kappa}\}$as the subgraph of G, which is formed by the n followers, where

$$\bar{A}=\begin{bmatrix}a_{11}&a_{12}&\cdots&a_{1n}\\\vdots&\vdots&\ddots&\vdots\\a_{n1}&a_{n2}&\cdots&a_{nn}\end{bmatrix}\in R^{n\times n} \tag{7}$$

Let$\bar{D}=\operatorname{diag}\{\bar{d}_1,\bar{d}_2,\cdots,\bar{d}_n\}\in R^{n\times n}$be a diagonal matrix with $\bar{d}_i=\sum_{j=1}^{n}a_{ij}$for $i=1,2,\cdots,n$. Then, it is clear that the Laplacian of the graph $\bar{G}$can be defined as

$$\bar{L}=\bar{D}-\bar{A} \tag{8}$$

The connection weight between agent i and the leader is denoted by

$$\bar{B}=\operatorname{diag}\{b_1,b_2,\cdots,b_n\} \tag{9}$$

In this paper, the following assumptions are considered.

Assumption 1. The time-varying control input u_0 is unknown to any follower but its upper bound $\bar{u}_0$is available to its neighbors.

Assumption 2. The position of the leader x_0 and its velocity v_0 are available to its neighbors only.