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Practice article

Synchronization of different fractional order chaotic systems with time-varying parameter and orders

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ABSTRACT

In this paper, synchronization of time-varying fractional order chaotic systems, is introduced. Parameters of system play an important role in chaotic systems. A time-varying parameter is selected for chaotic systems, also orders of systems are considered as time-varying orders. A reliable controller is designed to synchronize two different fractional order chaotic systems based on Lyapunov stability theory on fractional order systems. Numerical simulations and practical implementation of proposed method are presented to verify the results.

1. Introduction

Generalization of the traditional calculus is known as fractional calculus. Although fractional calculus has 300 years history until recent decades, it is noticed in many research fields [1]. Many systems are modeled with fractional-order dynamics [2–4]. For the first time, existence of chaos in fractional order systems is studied by Grigorenko and Grigorenko in 2003 [5]. After that many other fractional order chaotic systems are introduced, such as fractional order Chua's system [6], fractional order Lorenz system [7], fractional-order financial system [8], fractional order Duffing system [9] and fractional order Liu system [10].

One of the most important fractional chaotic systems is the fractional unified chaotic system [11] and fractional order electronic chaotic oscillator [12]. Unified chaotic system is a chaotic system which changes between Lorenz, Lü and Chen chaotic systems family with parameter changes. Synchronization of fractional order chaotic systems is one of the most important applications of chaotic systems.

Many effective methods have been introduced to achieve the synchronization between identical or different fractional-order chaotic systems, such as Pecora-Carroll principle [15], adaptive synchronization [16], observer-based synchronization [17], backstepping control [18], linear control [19] and sliding mode control [20]. One of the fractional order chaotic systems is fractional order electronic chaotic oscillator which has been introduced in Ref. [12] and chaotic behavior of this system has been shown for different parameter and order in this paper. Another well-known fractional order chaotic system is fractional order unified system which has been introduced in Ref. [11]. According to the completely different behavior, chaos synchronization between two different systems, always is a challenging problem. To the best of authors' knowledge, there are not any specific study on the synchronization problem of different time-varying chaotic systems, yet. There are some studies on synchronization of fractional order unified system [13–15], but these methods are used for the same fractional order unified systems with certain parameter and orders. However these methods are not usable methods when parameter of system changes in time domain. Also orders of a chaotic fractional order system, have important role in behavior of these systems. Then chaos synchronization in fractional order systems with time-varying orders makes the synchronization problem with more complexity. In this paper we introduce new methods for synchronization of time-varying fractional chaotic system. These variations are on parameter and orders of systems. We consider two different fractional time-varying systems then we propose suitable controllers for synchronization of these systems. First, based on fractional Lyapunov stability theory, controllers are designed. Then to reduce the control effort and synchronization time, optimal control conditions for time varying fractional order systems are considered and fractional time varying controllers are designed, also practical implementation of the proposed method has been done.

This paper is organized as follows. The basic definitions of fractional calculus and stability theorem of fractional order systems and some results of this theorem are introduced in Section 2. In Section 3, we introduce the time-varying fractional order chaotic systems and practical implementation of these systems. In section 4 we discuss the synchronization problem of fractional-order systems. In Section 5, we use numerical simulations, to show advantages of the proposed method. Finally, Section 6 concludes this paper.

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2. Fractional calculation

2.1. Fractional operator definitions

There are some definitions for fractional order derivate [21–23]. Three most popular definitions are described in follows.

2.1.1. Grünwald-Letnikov fractional derivative

In Grünwald-Letnikov (GL) definition, fractional derivate of a continuous function is represented as

$$D^{\alpha}x(t)=\lim_{h\to 0}h^{-\alpha}\sum_{j=0}^{(t-\alpha)/h}(-1)^{j}\binom{\alpha}{j}x(t-jh) \tag{1}$$

Where α is fractional order and $\binom{\alpha}{j}=\frac{\alpha(\alpha-1)\ldots(\alpha-j+1)}{j!}$.

2.1.2. Riemann-Liouville fractional derivative

Riemann-Liouville (RL) definition of fractional order derivate is different from (GL) definition. In this definition fractional order derivate is as follows.

$$D^{\alpha}x(t)=\frac{1}{\Gamma(n-\alpha)}\frac{d^{n}}{dt^{n}}\int_{0}^{t}\frac{x(\tau)}{(t-\tau)^{\alpha+1-n}}d\tau \tag{2}$$

where n is the first integer number bigger than α and Γ is the Gamma function.

2.1.3. Caputo fractional derivative

Caputo definition of fractional order derivate has some similarity to RL definition but in the general form it is described as

$$\frac{d^{\alpha}}{dt^{\alpha}}x(t)=\frac{1}{\Gamma(n-\alpha)}\int_{0}^{t}\frac{x^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}}d\tau \tag{3}$$

Where n and Γ are as mentioned in the RL definition.

2.2. Stability of fractional order systems

Consider the following fractional order equation

$$\frac{d^{\alpha}x}{dt^{\alpha}}=f(x) \tag{4}$$

where $x=[x_1,x_2,\ldots,x_n]^T\in R^n$ is the states vector of system and $f(x)=[f_1(x),f_2(x),\ldots,f_n(x)]^T$ defines the system, also suppose $0<\alpha\le 1$. In this condition the following relations are introduced.

2.2.1. Stability analysis using integer version of system

Lemma 1. *For the system (4) with fractional order α, the Jacobian matrix at the equilibrium points of system (4) is defined as $J=\partial f(x)/\partial x$. If all of the eigenvalues of this matrix satisfy the following condition* [24–26]

$$|\arg(eig(J))|>\alpha\pi/2 \tag{5}$$

This region is shown in Fig. 1. Then, system (4) is locally asymptotically stable.

For $\alpha=1$, system (4) changes to integer version which can be written as

$$\frac{dx}{dt}=f(x) \tag{6}$$

Remark 1. *According to Lemma 1, for integer order system (6), stability condition is determined as*

$$|\arg(eig(J))|>\pi/2 \tag{7}$$

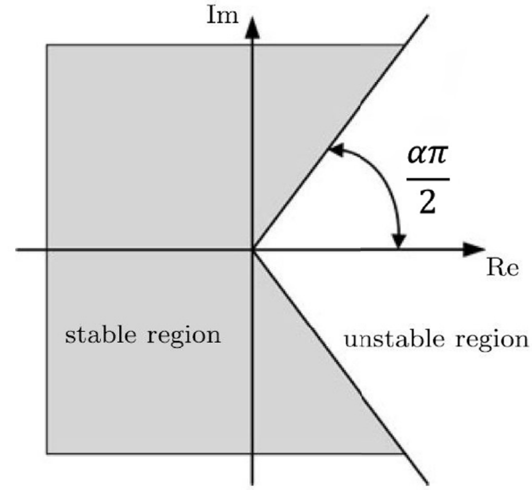

Fig. 1. Stability region of system (4) with $0<\alpha<1$.

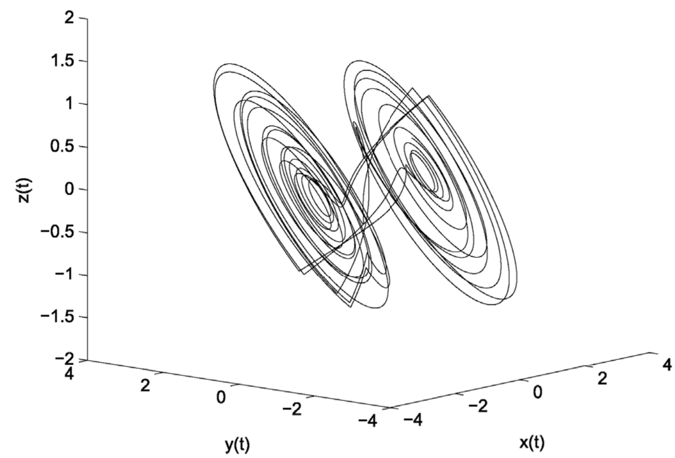

Fig. 2. Chaotic attractor of fractional-order ECO system with $\alpha=0.95$ and $a=0.55$.

2.2.2. Direct stability analysis for fractional time-varying system

Theorem 1. *For fractional order system (4) with equilibrium point as $x=0$, if there was a Lyapunov function $V(x(t))$ which satisfies* [31]

$$\alpha_1(\|x(t)\|)^m\le V(x(t))\le\alpha_2(\|x(t)\|) \tag{8}$$

and

$$\dot{V}(x(t))\le\alpha_3(\|x(t)\|) \tag{9}$$

where α_1, α_2, α_3 and m are positive constants. Then system (4) is Miattag-Leffler stable stable.

Theorem 2. *For fractional order system (4) with equilibrium point as $x=0$, if there was a Lyapunov function $V(x(t))$ which satisfies* [29]

$$\beta_1(\|x(t)\|)\le V(x(t))\le\beta_2(\|x(t)\|) \tag{10}$$

and

$$D^{\alpha}V(x(t))\le\beta_3(\|x(t)\|) \tag{11}$$

where β_1, β_2, β_3 are the class-K functions and $\alpha\in(0,1)$. Then system (4) is asymptotically stable.

Lemma 2. *For any continuous and derivable function x(t) and $\alpha(t)\in(0,1)$, we have*

$$\frac{1}{2}D^{\alpha(t)}x^2(t)\le x(t)D^{\alpha(t)}(x(t)) \tag{12}$$

Proof 1. *With definition (3) for $\alpha=1$, $n=1$ consider the right side of*