Weight restrictions in Data Envelopment Analysis: A comprehensive Genetic Algorithm based approach for incorporating value judgments

Vipul Jain [b], Amit Kumar [a], Sameer Kumar [c,*], Charu Chandra [d]

[a] Sapient Consulting Private Ltd., India
[b] Department of Industrial Engineering and Engineering Management, College of Engineering, University of Sharjah, P. O. Box 27272, Sharjah, United Arab Emirates
[c] Department of Operations and Supply Chain Management, Opus College of Business, University of St. Thomas, 1000 LaSalle Avenue, Minneapolis, MN 55403-2005, USA
[d] University of Michigan – Dearborn, College of Business Administration, FCS 183, 19000 Hubbard Drive, Dearborn, MI 48126-2638, USA

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ABSTRACT

The basic DEA model experiences the weights flexibility problem which is resolved by the method of weight restrictions. The current research incorporating Decision Makers' (DMs) preferences into weight restrictions is subject to serious limitations such as lacking a framework for dual role factors and not incorporating organizational hierarchy in decision-making.

The proposed Genetic Algorithm (GA) based approach for weight restrictions incorporates a dual role factor and organizational hierarchy in decision-making. The approach involves finding a set of weights which are at a minimum distance from all the DMs' preferences. The approach is flexible and is able to generate a common set of weights and Decision Making Unit (DMU) specific weight restrictions simultaneously.

Results from model validation in a well-known automobile spare parts manufacturer in India indicate that the majority of suppliers perceived as highly efficient were actually found to be inefficient in the GA based weight restrictions model.

A major contribution of this study is a robust approach to deal with multiple DMs and DEA weights flexibility problem. Another key highlight of the research is translating DMs preferences into a distance function. Using that as a fitness measure within the proposed Evolutionary Algorithms has been done for the first time in the presence of multiple DMs.

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1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric, linear programming based method for measuring the relative efficiency of a set of DMUs. However, the basic DEA approach proposed by Charnes, Cooper and Rhodes (CCR) suffers from the weights flexibility problem (Charnes, Cooper, & Rhodes, 1978). It allows total flexibility in weight selection for the DMUs in order to achieve maximum efficiency. This flexible choice of weights in CCR approach can be inconsistent with management views or prior knowledge of inputs and outputs.

Weight restrictions provide the solution to the weight flexibility problem in DEA. There are a number of studies on weight restrictions and how to incorporate value judgments into them.

Allen, Athanassopoulos, Dyson, and Thanassoulis (1997) and Thanassoulis, Portela, and Allen (2004) conclude that there is no all purpose method for translating value judgments into DEA weight restrictions. Some of the most prominent weight restriction methods such as Cone-Ratio (CR) models and Assurance Region (AR) models incorporate *a priori* information. Angulo-Meza and Lins (2002) point out such value judgments can be biased with ideas not being consistent with reality. Similar observations have been made by Allen et al. (1997). Approaches like Assurance Region II (AR II) are known to have feasibility issues (Allen et al., 1997).

The majority of these approaches deal with only a single Decision Maker (DM). There is one approach by Talaue, Diesta, and Tapia (2011) which deals with multiple DMs. The approach is not guaranteed to find a solution which would satisfy all DMs. Also, the approach assumes that all DMs are at par with each other. The current organizational structures have a hierarchy which would lead to certain DMs being senior to others and thus having more weight in decision making.

None of the approaches deal with weight restrictions on dual role factors in DEA. This is very important for a DEA approach

* Corresponding author.
E-mail addresses: Vjain@sharjah.ac.ae (V. Jain), amit.hooda.1511@gmail.com (A. Kumar), sameerkumar724@gmail.com (S. Kumar), charu@umich.edu (C. Chandra).

which deals with certain specific problems such as supplier selection as highlighted in Saen (2010).

Most of these approaches either try to find a common set of weights or DMU specific weight restrictions, but not both. Premachandra (2001) and Makui, Alinezhad, Mavi, and Zohrebandian (2008) suggest approaches for a common set of weights. Wong, Mariano, and Jian-Bo (2009) and Talaue et al. (2011) present approaches for DMU specific weight restrictions. However, none of these approaches incorporate both types of weight restrictions.

Some of the latest works in the area of weights restrictions use Assurance Regions (Halkos, Tzeremes, & Kourtidis, 2014; Liu, 2014) in a 2-stage DEA model. Another approach by Hosseinpour, Pourmahmoud, and Masrouri (2013) use symmetric weights in a DEA-AHP hybrid to overcome the weights flexibility issue. Mecit and Alp (2013) provide an approach in which weights are defined as correlation between inputs and output variables which provides a more balanced weights distribution. Hajiagha, Hashemi, and Mahdiraji (2014) present a Linear Programming based approach to calculate the common set of weights. A key drawback of all these latest approaches is that none of them attempt to translate the DM's preferences into weights restriction and hence, lack a framework for industrial application of such approaches. None of these can deal with multiple DMs which is quite often a case in today's industrial scenario. In addition, none of the approaches guarantee validity and feasibility in presence of dual role factor in DEA. A brief review of these approaches is presented in the next section.

To sum up, most weights restriction approaches tend to be lacking in one or more of the following:

- **Managérial Implications** – One of the most desirable properties in an approach for translating value judgements would be to offer an "easy to translate" framework which would allow DMs to come forward with a more quantitative view of their judgments.
- **Subjectivity Concerns** – Some of the most prominent weight restriction methods incorporate *a priori* information which reduces discrimination in DEA.
- **Feasibility** – Some of these approaches do not guarantee feasibility.
- **Multiple DMs** – The majority of these approaches deal with only a single Decision Maker (DM).
- **Dual Role Factors** – None of the approaches deal with weight restrictions on dual role factors in Data Envelopment Analysis
- **Comprehensiveness** – Most of these approaches either try to find a common set of weights or DMU specific weight restrictions, but not both.

Research Goal: This paper proposes a Genetic Algorithm (GA) based approach to overcome all the above limitations. The approach takes into account feasibility, and can find a common set of weights and DMU specific weight restrictions at the same time. The approach is based on finding a set of weights which are at a minimum distance from all the DMs' preferences. Thus the proposed weight restrictions would be a feasible set which would be closest to all of the DMs' preferences. The GA takes into account dual role factors and ensures feasibility at all times. The distance function eliminates the subjectivity concerns and also provides an "easy to use" framework for the DMs.

The GA based approach is applied to the supplier selection process for a well-known automobile spare parts manufacturer in India and the results are presented.

The rest of the paper is structured as follows. Section 2 presents related work and an outline of proposed GA based weight restrictions approach. Section 3 describes theoretical details associated with GA based weight restrictions approach. Section 4 shows a practical application of the proposed theoretical approach showing data and validation of the proposed approach. Section 5 reports results, a comparison of the proposed approach with AHP/ANP approaches and discussion in terms of significance of the results from the application of GA based approach. Finally, Section 6 presents the conclusions, implications and limitations of the study.

2. Material and methods

There have been numerous research studies on solving the weight flexibility problem in DEA. Weight restrictions were first imposed by Dyson and Thanassoulis (1988) in an attempt to incorporate top management perspectives on the relative importance of the inputs and outputs used in the assessment. Charnes, Cooper, and Huang (1990) presented the cone ratio approach which is based on pre-selection of DMUs or favored inputs/outputs. However, pre-selection of DMUs may not be a good approach as pointed out by Thanassoulis, Dyson, and Foster (1987) in their rates department study.

Thompson, Singleton, Thrall, and Smith (1986) presented the concept of Assurance Regions (ARs) which was based on imposing homogeneous linear restrictions. The approach incrementally refines an Assurance Region until the efficiency levels are satisfactory to the Decision Maker. This type of approach is called AR I. AR II was introduced by Thompson et al. (1986) and used by Thanassoulis et al. (2004) which imposed restrictions on the ratio between input and output weights.

A review of various weight restriction methodologies and their evolution is presented in Allen et al. (1997) and Thanassoulis et al. (2004). These two articles present the fallout of the traditional cone ratio approach and Assurance Region models in terms of parameter estimation to be used for DEA weight restrictions.

There are number of approaches in literature which use AHP and ANP to translate the DMs' value judgments into weight restrictions (Guo, Jia, & Qiu, 2006; Premachandra, 2001; Royendegh & Erol, 2009; Sinuany-Stern, Mehrez, & Hadad, 2000; Zhang, Li, & Liu, 2006). While these approaches are appropriate for weight restriction, these approaches suffer from various limitations. For instance, AHP and ANP centered approaches are based on pair-wise comparisons which would essentially imply that these approaches are limited to relative weight restrictions only. With multiple DMs, AHP can allow for powerful DMs or outliers in DMs' judgments to influence the decision making process significantly. Palcic and Lalic (2009) observed this behavior in their study.

Hosseinpour et al. (2013) present the symmetric weights base approach for an AHP-DEA hybrid. The approach improves on an existing AHP-DEA hybrid which had the drawback of not preserving rankings. Mecit and Alp (2013) present a DEA weights approach in which weights are expressed as input–output ratios. The results demonstrated that such correlation based approaches can achieve a balanced weight distribution unlike other known DEA models which tend to have a skewed distribution. The numerical examples in both the studies are from existing literature and an end-to-end industrial application of the approaches is lacking.

Liu (2014) present the Assurance Region approach for 2-stage DEA in presence of fuzzy input–output data. An example from non-life insurance companies in Taiwan is presented to validate the approach in presence of fuzzy data. Halkos et al. (2014) also propose an Assurance Region based approach for 2-stage DEA. The approach is validated across secondary education institutes from 65 countries to construct an overall school efficiency index. However, both these approaches do not deal with multiple DMs or provide any insight on translating DMs opinions into weights restrictions.

The optimization problem that this study attempts to solve is formulated as below: