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The super connectivity of exchanged crossed cube ☆

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ABSTRACT

Li et al. proposed a new interconnection network, named exchanged crossed cube $ECQ(s,t)$, which has better properties than other variations of hypercube in the field of the fewer diameter, smaller links and lower cost factor. This work will show that its super connectivity and super edge-connectivity are both equal to $2s$ with $s \leq t$. It means that at least $2s$ vertices (resp. $2s$ edges) of $ECQ(s,t)$ are moved away to obtain a disconnected graph avoiding isolated vertex.

1. Introduction

Throughout this work, a graph $G = (V, E)$ always means a simple undirected graph, where $V = V(G)$ and $E = E(G)$ are the vertex set and the edge set of G respectively. Let $v \in V$ and $M \subseteq V$. Then $N(v)$ is the neighborhood of v in G and $d(v) = |N(v)|$ is the degree of v in G. Put $N_M(v) = N(v) \cap M$. We use $G[M]$ to represent the subgraph of G induced by M. $\delta(G)$ is the minimum degree of G. For the terminology and notation not defined here, we refer to [2].

A (vertex) cut S (resp. edge cut) of G is a vertex (resp. edge) set of G such that $G - S$ is disconnected or trivial. The (vertex) connectivity (resp. edge-connectivity) of G, denoted by $\kappa(G)$ (resp. $\lambda(G)$), is an important concept in network, which is the minimum size over all the (vertex) cuts (resp. edge cuts) of G. As more refined indices, Esfahanian and Hakimi [5] introduced restricted connectivity (resp. restricted edge-connectivity) by considering cuts (resp. edge cuts) that satisfy some conditions. Based on this concept, super connectivity and super edge-connectivity were proposed in [1]. A graph G is super connected (resp. super edge-connected) if each minimum order cut (resp. edge cut) isolates a vertex of G. A natural problem is that how many vertices or edges must be deleted to disconnect G such that each component of the remaining graph contains no isolated vertices. This results in the following concept.

Letting $F \subseteq V(G)$ (resp. $F \subseteq E(G)$), F is called a super vertex cut (resp. super edge-cut) of G if $G - F$ is disconnected and every component has at least two vertices. Super vertex cuts or super edge-cuts do not always exist. The super connectivity (resp. super edge-connectivity) of G, denoted by $\kappa'(G)$ (resp. $\lambda'(G)$), is the minimum cardinality over all super vertex cuts (resp. super edge-cuts) if there is any, and is $+\infty$ otherwise by convention. Many studiers focus their attention on super connectivity and super edge-connectivity of networks, such as [4,6,7,10–12,14–19].

The hypercube is widely known as one of the most popular interconnection networks for parallel computing systems. Basing on two important interconnection network exchanged hypercube $EH(s,t)$ [9] and crossed cube CQ_n [3], Li et al. [8] proposed a new interconnection net-

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work exchanged crossed cube $ECQ(s,t)$. Just as $EH(s,t)$ and CQ_n, $ECQ(s,t)$ is also a variant of hypercube. This interconnection topology retains most of the topological features of $EH(s,t)$, and meanwhile combines many attractive properties of CQ_n. It has been proved that $\kappa(ECQ(s,t)) = \lambda(ECQ(s,t)) = s+1$ with $s \le t$ [13]. Here we show that $\kappa'(ECQ(s,t)) = \lambda'(ECQ(s,t)) = 2s$ with $s \le t$.

2. Exchanged crossed cube

In order to define $ECQ(s,t)$, we show the following necessary auxiliary definition.

Definition 2.1. Two binary strings $x = x_1x_0$ and $y = y_1y_0$ are pair related, denoted by $x \sim y$, if and only if $(x,y) \in \{(00,00),(10,10),(01,11),(11,01)\}$.

In [8], Li et al. defined $ECQ(s,t)$ as follows.

An $ECQ(s,t)$ with positive integers $s \ge 1$ and $t \ge 1$ is defined as an undirected graph. Its vertex set V is

$$V = \{a_{s-1}\cdots a_0 b_{t-1}\cdots b_0 c | a_j, b_i, c \in \{0,1\} \text{ for } j \in [0,s-1], i \in [0,t-1]\}.$$

For a vertex $v = a_{s-1}\cdots a_0 b_{t-1}\cdots b_0 c$, the dimension of c, b_i, a_j are $0, i+1, t+j+1$ respectively. Meanwhile $v[k]$ represents the value of the dimension k in v.

The edge set $E = \{(v_1,v_2)|(v_1,v_2) \in V \times V\}$ consists of disjoint parts E_1, E_2 and E_3.

$E_1: v_1[0] \ne v_2[0]$, $v_1 \oplus v_2 = 1$, where $\oplus$ is the exclusive-OR operator.
$E_2: v_1[s+t:t+1] = v_2[s+t:t+1]$, $v_1[0] = v_2[0] = 1$, $v_1[t:1]$ denoted by $b = b_{t-1}\cdots b_0$ and $v_2[t:1]$ denoted by $b' = b'_{t-1}\cdots b'_0$ are joined according to the following rule: For all $t \ge 1$, if and only if there exists an l $(1 \le l \le t)$ with (1) $b_{t-1}\cdots b_l = b'_{t-1}\cdots b'_l$, (2) $b_{l-1} \ne b'_{l-1}$, (3) $b_{l-2} = b'_{l-2}$ if l is even, and (4) $b_{2i+1}b_{2i} \sim b'_{2i+1}b'_{2i}$ for $0 \le i < \lfloor (l-1)/2 \rfloor$.

$E_3: v_1[t:1] = v_2[t:1]$, $v_1[0] = v_2[0] = 0$, $v_1[s+t:t+1]$ denoted by $a = a_{s-1}\cdots a_0$ and $v_2[s+t:t+1]$ denoted by $a' = a'_{s-1}\cdots a'_0$ are joined according to the following rule: For all $s \ge 1$, if and only if there exists an l $(1 \le l \le s)$ with (1) $a_{s-1}\cdots a_l = a'_{s-1}\cdots a'_l$, (2) $a_{l-1} \ne a'_{l-1}$, (3) $a_{l-2} = a'_{l-2}$ if l is even, and (4) $a_{2i+1}a_{2i} \sim a'_{2i+1}a'_{2i}$ for $0 \le i < \lfloor (l-1)/2 \rfloor$.

Here $v[x:y]$ is the bit pattern of v from dimension y to dimension x. It is easy to verify that $ECQ(s,t)$ has 2^{s+t+1} vertices. For simplicity, we sometimes use ECQ instead of $ECQ(s,t)$. Fig. 1 shows $ECQ(1,3)$.

In [8], authors announced some fundamental properties of ECQ.

Theorem 2.1. *(See [8].) $ECQ(s,t)$ is isomorphic to $ECQ(t,s)$.*

Hence we can always assume $s \le t$ hereafter. Then $\delta(ECQ(s,t)) = s+1$.

Theorem 2.2. *(See [8].) $ECQ(s,t)$ can be partitioned into two copies of $ECQ(s-1,t)$ or $ECQ(s,t-1)$.*

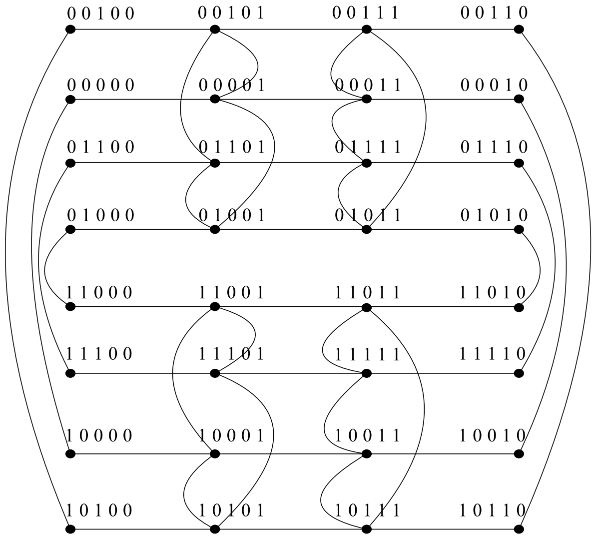

Fig. 1. $ECQ(1,3)$.

For the connectivity of ECQ, there has the following theorem.

Theorem 2.3. *(See [13].) $\kappa(ECQ(s,t)) = \lambda(ECQ(s,t)) = s+1$ with $s \le t$.*

In this work, we will show the following main result on super connectivity.

Theorem 2.4. *$\kappa'(ECQ(s,t)) = 2s$ with $s \le t$.*

3. Proof

Proof of Theorem 2.4. ECQ can be partitioned into subgraphs L and R, where

$$V(L) = \{0a_{s-2}\cdots a_0 b_{t-1}\cdots b_0 c | a_j, b_i, c \in \{0,1\} \text{ for } j \in [0,s-2], i \in [0,t-1]\},$$

$$V(R) = \{1a_{s-2}\cdots a_0 b_{t-1}\cdots b_0 c | a_j, b_i, c \in \{0,1\} \text{ for } j \in [0,s-2], i \in [0,t-1]\}.$$

L and R are the subgraphs induced by $V(L)$ and $V(R)$ respectively. Obviously L and R are both isomorphic to $ECQ(s-1,t)$. Put

$$A = \{0a_{s-2}\cdots a_0 b_{t-1}\cdots b_0 1 | a_j, b_i \in \{0,1\} \text{ for } j \in [0,s-2], i \in [0,t-1]\},$$

$$B = \{0a_{s-2}\cdots a_0 b_{t-1}\cdots b_0 0 | a_j, b_i \in \{0,1\} \text{ for } j \in [0,s-2], i \in [0,t-1]\},$$

$$C = \{1a_{s-2}\cdots a_0 b_{t-1}\cdots b_0 0 | a_j, b_i \in \{0,1\} \text{ for } j \in [0,s-2], i \in [0,t-1]\},$$

$$D = \{1a_{s-2}\cdots a_0 b_{t-1}\cdots b_0 1 | a_j, b_i \in \{0,1\} \text{ for } j \in [0,s-2], i \in [0,t-1]\}.$$

By this partition, we can obtain the following properties directly (see Fig. 2).