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Compromise utilitarian solutions in multi-criteria optimization problems as a guide for evolutionary algorithms

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ABSTRACT

Multi-criteria optimization problems are considered where the decision maker is unable to determine the exact weights of importance of the criteria but can provide some imprecise information about these weights. Two solution concepts are studied in this framework: the *optimistic min–max solution* and the *compromise utilitarian solution*, both of which can be exactly computed for linear problems. For general problems, it is shown that these solutions can be approximated by means of a slight modification of the evolutionary algorithm NSGA-II.

1. Introduction

Egalitarianism and utilitarianism are two axiomatic principles which represent the preferences of the decision maker (DM), upon which many approaches to multi-criteria optimization problems (MOPs) are based. When using an additive value function that involves all the criteria scores for the selection of a decision, a utilitarian principle is being applied, whereas egalitarianism is the motivation for the use of min–max solutions (see [16]). In both cases, the weights of importance of the criteria play a leading role.

Very often the DM may be doubtful when setting precise values for criteria weights. In the model presented in this paper, imprecision on criteria weights is allowed and the information about these weights is formalized by means of linear constraints. Thus, a polyhedron of weights of importance of the criteria is considered instead of a single vector of weights. This kind of information is often designated as incomplete, imprecise or partial information.

Previous related studies on the various treatments of MOPs with partial information are fundamentally concerned with the reduction of the set of Pareto-optimal solutions according to the available information (see for example [2,12,18]). There are also certain results on multi-criteria linear problems in which the coefficients of the objective function are not precisely defined, but are given by intervals or by linear relations (see [11,9,19,20]). Most of these approaches focus on the analysis of the sensitivity of a solution given feasible changes in the parameters. In a different way, in [10], the solutions of multi-objective decision problems with partial information are studied when the DM's preferences are represented by utilitarian or egalitarian functions and the rationality principles that sustain them are analyzed.

In this paper, we investigate the solutions which arise when the DM's preferences are represented by utilitarian and egalitarian value functions in the context of partial information. Since the straightforward extensions of the utilitarian and the min–max solutions generally produce too many Pareto-optimal outcomes, we propose further refinements in order to obtain solutions that permit the decision making process to be completed. By using an egalitarian principle, one of these possible refinements may consist of simply choosing the maximal of the polyhedron of weights of importance of the criteria and of considering the outcomes provided by the weighted min–max solution. Throughout the paper, these outcomes are called the *optimistic min–max solutions* (*OMS*).[1] Since the OMS takes egalitarianism to an extreme and there are situations where the egalitarian principle is inappropriate in the choice of the solutions (see Fig. 3 in Section 3.2), in this paper compromise outcomes are proposed between the OMS and the outcomes arising from the corresponding utilitarian solution. We call the outcomes arising from this compromise the *compromise utilitarian solutions* (*CUS*).

Although the CUS and the OMS can easily be computed for multi-criteria continuous linear problems, they are difficult to compute in general. Nevertheless, we show how evolutionary multi-objective optimization (EMO) algorithms can successfully

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[1] Later on, the reason for which it is called "optimistic" will be explained.

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approximate these solutions. The success of the procedure is due: to the ability of EMO algorithms to quickly solve multi-objective problems (or compute a good approximation of the solution set); to their robustness (although they are stochastic processes); and to their flexibility to manage constraints that represent the preferences of the DM. In addition, the use of an EMO algorithm instead of a single-objective evolutionary algorithm is justified as follows: on the one hand, according to the uniqueness of the CUS and of the OMS (see Example 3.8), the use of an EMO algorithm allows practitioners to manage problems not knowing in advance if they have an unique or multiple solutions; on the other hand, EMO algorithms maintain a more diverse population and usually obtaining higher quality solutions.

Evolutionary multi-objective optimization algorithms were originally proposed to solve hard multi-objective optimization problems. Since in only one run they generate the true Pareto front (or a good approximation), they have been successfully employed in recent decades to solve real complex problems in a variety of research fields, such as engineering, biology, business, and telecommunications (see [4], [5]). Nevertheless, EMO algorithms present many other good properties that make them suitable to modify, for example, the search process of the algorithm and guide the population towards those preferred areas in the true Pareto front when a priori information is provided. To this end, several attempts have been made according to the kind of partial information available.

In [1], an excellent review of this topic is given through the classification of the approaches into several categories: when the articulation of the DM's preferences is carried out before (a priori), during (progressive), or after (a posteriori) the search. Other recent contributions related to this topic include: [14] where the partial information is provided as a (feasible or unfeasible) reference point to follow; [8] where the authors use a binary fuzzy preference relation; and [17] where a new set of decision rules is employed to obtain values for the scaling weights in each function.

The rest of the paper is organized as follows. In Section 2 the model is formalized, in Section 3 the notions of optimistic min-max solution and compromise utilitarian solution is introduced. Finally, Section 4 is devoted to show how an EMO algorithm (the NSGA-II is adopted in this work) can be modified in order to obtain a good approximation of the CUS (OMS) and some computational results are provided which validate this approximation.

2. The model

A multi-criteria optimization problem (MOP) with s criteria can be formally defined as

$$\begin{aligned} \min \quad & f(x) = [f_1(x), \ldots, f_s(x)] \\ \text{s.t.}: \quad & x \in X \end{aligned}$$

and represented by the pair (X,f), where $X \subseteq \mathbb{R}^n$ is the feasible set in the decision space, each $x \in X$ is an n-dimensional vector of *decision variables*, and $f : \mathbb{R}^n \to \mathbb{R}^s$ is a vector-valued function, whose s components, $f_1(x), \ldots f_s(x)$, are s scalar functions of the decision vector x which represent the *criteria* that have to be taken into account to evaluate each feasible decision.[2] Without loss of generality, we suppose that all the objectives are to be minimized.

Pareto optimality is a basic requirement in MOPs:

[2] Notice that we do not assume continuity neither in the feasible decision set (it may consist of a finite set of alternatives) nor in the objective function. Nevertheless, as compensability between the criteria is assumed, we consider that the valuations of feasible decisions are normalized in order to they can be compared.

Defnition 2.1. A feasible solution $x^* \in X$ is Pareto optimal (or efficient) if there is no other feasible solution, $x \in X$, such that $f_i(x) \le f_i(x^*)$ for all $i = 1,2, \ldots, s$ and $f(x) \ne f(x^*)$. A feasible solution $x^* \in X$ is weakly Pareto optimal (or weakly efficient) if there is no $x \in X$ such that $f_i(x) < f_i(x^*)$ for all $i = 1,2, \ldots, s$.

Given a *MOP*, (X, f), the Pareto optimal (weakly Pareto optimal) solution set is denoted by $PO(X, f)$, $(WPO(X, f))$.

Various procedures exist for the generation of efficient solutions and their selection depends on whether one is interested in generating one (or a few) Pareto optimal solution(s) or the complete (or a good approximation of the) Pareto frontier. In the former case, classic methods, which are focused on reducing the number of objective functions into a single auxiliary objective function, have been successfully applied over recent decades, and include the weighted sum method, ϵ-constraints, weighted metric methods, compromise programming, goal programming methods and many others (see [3,21,13]). In the latter case, the most extended approaches to generate the complete Pareto front in a single run are evolutionary multi-objective optimization algorithms, such as genetic algorithms, evolution strategies, and evolutionary programming (see [5,4] for a complete review of this topic).

In this paper, the latter approach is adopted, with the additional incorporation of imprecise information about the preferences of the decision maker in order to guide the search. This information consists of linear constraints on the weights of importance of the criteria.

The theoretical approach, which constitutes the starting point for the analysis presented in this paper, is established below: by using weights of importance of the criteria in the $\mathbb{R}^s$-simplex $\Delta^{s-1} = \{\lambda \in \mathbb{R}^s_+ \mid \sum_{i=1}^{s} \lambda_i = 1\}$, the vector valuation of each feasible decision can be reduced to a scalar valuation. When it is possible to determine an exact weighting vector, $\lambda \in \Delta^{s-1}$, two relevant solutions in the class of MOPs are the utilitarian and the min–max solutions. These are rationalized by an additive value function, $\sum_{i=1}^{s} \lambda_i f_i(x)$, and by the function $\max_{i=1,\ldots,s}\{f_i(x)/\lambda_i\}$, respectively.

Defnition 2.2. The utilitarian solution, U_λ, selects, for every problem (X,f), the set

$$U_\lambda(X,f) = \arg\min_{x \in X} \sum_{i=1}^{s} \lambda_i f_i(x).$$

Defnition 2.3. The min–max solution, M_λ, selects, for every problem (X,f), the set

$$M_\lambda(X,f) = \arg\min_{x \in X} \max_{i=1,\ldots,s} \left\{ \frac{f_i(x)}{\lambda_i} \right\}.$$

Remark 2.4. Note that the outcomes associated to the min–max multi-criteria solution can be obtained by solving a min–max problem which is equivalent to the following:

$$\left.\begin{aligned} \min \quad & t \\ \text{s.t.}: \quad & f_i(x) \le \lambda_i t, \quad i = 1,2, \ldots, s, \\ & x \in X \end{aligned}\right\} \tag{2.1}$$

If the weights associated to the criteria are identical, that is $\lambda_i = \lambda_j$ for all $i, j \in \{1,2, \ldots, s\}$, then the *utilitarian solution*, based on the L_1 metric,[3] and the *min–max solution*, based on the Tchebychev metric,[4] are obtained. The utilitarian solution exhibits no

[3] The L_1 distance between $z^1 \in \mathbb{R}^s$ and $z^2 \in \mathbb{R}^s$ is $d_1(z^1,z^2) = \sum_{i=1}^{s} |z_i^1 - z_i^2|$.

[4] The Tchebychev distance between $z^1 \in \mathbb{R}^s$ and $z^2 \in \mathbb{R}^s$ is $d_\infty(z^1,z^2) = \max\{|z_i^1 - z_i^2|, \ i = 1,2, \ldots, s\}$.