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Approximating multi-objective scheduling problems

Said Dabia [a,b,*], El-Ghazali Talbi [c], Tom van Woensel [b], Ton De Kok [b]

[a] Eyefreight B.V., the Netherlands
[b] Eindhoven University of Technology, School of Industrial Engineering, The Netherlands
[c] University of Lille, INRIA, CNRS, France

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ABSTRACT

In many practical situations, decisions are multi-objective by nature. In this paper, we propose a generic approach to deal with multi-objective scheduling problems (MOSPs). The aim is to determine the set of Pareto solutions that represent the interactions between the different objectives. Due to the complexity of MOSPs, an efficient approximation based on dynamic programming is developed. The approximation has a provable worst case performance guarantee. Even though the approximate Pareto set consists of fewer solutions, it represents a good coverage of the true set of Pareto solutions. We consider generic cost parameters that depend on the state of the system. Numerical results are presented for the time-dependent multi-objective knapsack problem, showing the value of the approximation in the special case when the state of the system is expressed in terms of time.
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1. Introduction

Many optimization problems encountered in practice are multi-objective by nature, i.e., different objectives are conflicting and equally important. Many times, it is not desirable to drop some of them or to optimize them in a hierarchical manner. For instance, while designing a product, many criteria are taken into account: the product's reliability should be maximized, while the cost and the environmental impact should be minimized.

Contrary to single-objective optimization problems where the optimal value is unique, the aim of multi-objective optimization problems (in short, MOPs) is to determine the set of solutions representing the trade-offs between the different conflicting objectives. This set of solutions is denoted as the set of Pareto solutions or the Pareto front. In this line of thought, decision makers are presented with the entire Pareto front (rather than a single solution) to select a solution (or a region of solutions) depending on their preferences. Although the roots of multi-objective optimization go back to the nineteenth century in the work of Edgeworth [3] and Pareto [24], most optimization problems dealt with are single-objective. In fact, objective functions are usually reduced to a composite single-objective function by using a (weighted) sum of the various objectives. It is argued that solutions obtained in this way might represent only a subset of the entire set of Pareto solutions, and therefore could lead to suboptimal managerial decisions [4,22,31].

In multi-objective decision making, the number of Pareto solutions increases with the size of the problem, mainly with the number of objectives. Therefore, multi-objective decision making is very challenging. In fact, multi-objective problems are $\mathcal{NP}$-hard. Hence, it is computationally expensive to compute the complete Pareto front. Furthermore, multi-objective decision making does not end when the Pareto front is found. In practice, only a single solution (or a region of solutions), taking decision makers preferences into account, needs to be implemented. There exist several methods allowing the selection of a solution from the Pareto front [7]. These methods might not converge easily when the size of the Pareto front is very large. Consequently, many researchers direct their efforts to approximating the Pareto front to reduce the complexity of the applied algorithms. Approximate Pareto fronts contain fewer solutions, which facilitate the selection of a final solution. However, a good approximate Pareto front should properly represent the real Pareto front.

In this paper, an approximation algorithm based on dynamic programming is proposed for *multi-objective scheduling problems* (MOSPs). The multi-dimensional state space is partitioned into intervals with exponentially increasing size. Each interval defines a cluster of states considered to be *very close* to each other. From each cluster, only one state is kept and the dynamic programming is adapted to the partitioned state space. In this way, in each iteration of the dynamic programming, only a polynomial number of states is processed. The approximation has a provable performance guarantee. Even with fewer solutions, the resulting approximate Pareto front still properly covers the real Pareto front as each

* Corresponding author at: Eindhoven University of Technology, School of Industrial Engineering, The Netherlands. Tel.: +31 402472081.
E-mail addresses: s.dabia@eyefreight.com (S. Dabia), El-ghazali.Talbi@lifl.fr (E.-G. Talbi), t.v.woensel@tue.nl (T. van Woensel), a.g.d.kok@tue.nl (T. De Kok).

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Pareto solution is represented by at least one approximate Pareto solution. The proposed approximation can be applied to the multi-objective version of a variety of well-known optimization problems for which a dynamic programming formulation is possible (e.g., knapsack problems, shortest path problems, variants of vehicle routing problems, job scheduling problems, etc.). Furthermore, we consider a generic cost structure where costs depend on the state of the system. In many practical situations, cost parameters are not constant. In the transport sector, for instance, carriers work with tariff sheets where costs are computed depending on the utilization of their fleets. In fact, the tariff depends on the truck load or on the total kilometers traveled. We show the value of the approximation by applying it to the time-dependent multi-objective knapsack problem where the state of the system is expressed as a function of time.

The contributions of this paper are summarized as follows. A generic approximation is proposed which can be applied to multi-objective scheduling problems for which a dynamic programming formulation is possible. The approximation generates an approximate Pareto front with fewer solutions. The approximate Pareto front represents a very good coverage of the real Pareto front. Additionally, the approximation's worst case performance guarantee is provable. The approximation is flexible in the sense that the decision maker can choose different precision levels for the different objectives. In fact, the decision maker might be willing to tolerate more error for less sensitive objectives (i.e., with a "flat" cost structure). Finally, this paper deals with a class of realistic MOSPs for which costs are state-dependent. For instance, in a traffic network, travel costs are a function of travel times which change depending on the state of the traffic network (e.g., due to congestion).

This paper is organized as follows. Section 2 reviews the literature relevant to MOPs. Section 3 is devoted to the introduction of the main concepts related to MOPs. Section 4 describes a generic MOSP, the input structure and the dynamic programming formulation for the MOSP. In Section 5, an approximation of the Pareto front is developed and the main results of the paper are derived. In Sections 6 and 7, the methodology developed in this paper is validated on the time-dependent multi-objective knapsack problem. Finally, Section 8 concludes with a summary of the main results.

2. Selected literature review

As in single-objective optimization, MOPs can be divided into two categories: problems with *real-valued* variables, also known as *continuous MOPs*, and problems with *discrete* variables, known as multi-objective combinatorial optimization problems (*MOCO*). In the class of continuous MOP, usually an infinite number of Pareto solutions composes the Pareto front, whereas in combinatorial MOPs, the Pareto front is finite. Most heuristics for solving MOPs are designed to deal with continuous MOPs using, for instance, multi-objective simplex [37,29]. In the last decade, there is a growing interest in solving combinatorial MOPs. However, in most cases, they are bi-objective optimization problems. Furthermore, there is a lack of test instances for real-life combinatorial MOPs, especially problems with many objectives [11,19] and dynamicity [6].

The study of computational complexity classes for MOCO started with the work of Serafini [28], and Papadimitriou and Yannakakis [23], where a connection was made between the results obtained in single-objective combinatorial optimization and the multi-objective field for several optimization problems. Serafini [28] depicts nine possible definitions for MOCO problems and establishes reductions between them in order to facilitate obtaining complexity results. He shows that the following definition (denoted as V8 in his article) can be considered as a standard reference version to measure the computational complexity of MOCO problems. The definition can also be seen as the decision problem associated with a MOCO problem.

Definition 1 (*Generic definition of MOCO by Serafini [28]*). Given $z \in Z^n$, does there exist a solution x to MOCO such that $g_i(x) \le z_i, 1 \le i \le n$?

Where the functions g_i reflect some measures of interest, and $g_i(x)$ is computable in polynomial time. An $\mathcal{NP}$-hard single-objective problem implies an $\mathcal{NP}$-hard character to its multi-objective extensions. In the multi-objective case, the $\mathcal{NP}$-hardness appears for the majority of problems. For example, $\mathcal{NP}$-hardness is proved for shortest path problems, assignment problems and minimum maximal matching by Serafini [28]; for the minimum weight spanning tree by Camerini and Vercellis [1]; and for the max-linear spanning tree by Hamacher and Ruhe [8].

Similarly to single-objective optimization problems, MOPs can be solved by means of *exact* and *approximate* algorithms. In the literature, more attention has been devoted to bi-criteria optimization problems by using exact methods such as branch-and-bound algorithms [27,32,33,26,17], branch-and-cut [12], *A* algorithm* [30,20], and dynamic programming [34,2]. There exist some new advances in this area, with several exact approaches proposed in the literature for bi-objective [16,14,18] and multi-objective problems [16]. Approximate methods are mainly used to solve large-scale problems and when multiple criteria are involved. They can be divided into two classes. On the one hand, there are algorithms that are only applicable to a specific problem. Such algorithms are developed based on some knowledge on the structure of the problem at hand. On the other hand, we see metaheuristics which are of general purpose, in the sense that they can be applicable to a large variety of MOPs. A unifying view for analyzing, designing and implementing multi-objective metaheuristics is provided in the book by Talbi [31]. The main drawback of metaheuristics is the lack of guarantee with regard to the quality of the approximate Pareto front. Moreover, the resulting approximate Pareto fronts might not properly cover the real Pareto front as they might contain very few solutions.

In the context of single-objective optimization problems, an ϵ-approximation scheme is an algorithm that, for every instance of the problem, finds an approximate solution that is guaranteed to be within a random constant factor from optimal. Two classes of approximation schemes are mainly considered: polynomial time approximation scheme (PTAS) and fully polynomial time approximation scheme (FPTAS). For any $\epsilon > 0$, a PTAS has run-time which is polynomial in the size of the instance, while an FPTAS has run-time which is polynomial in the size of the instance and $1/\epsilon$. From a computational complexity point of view, FPTASs are the strongest approximation schemes with performance guarantee that can be obtained for $\mathcal{NP}$-hard optimization problems. The notion of approximation schemes can be generalized to the case of multi-objective optimization problems by considering, for each solution on the approximate Pareto front, worst case performance guarantees with regard to all criteria [5]. In Legriel et al. [15] and Marinescu [21], approximation methods for multi-objective optimization problems are provided. However, the complexity of the presented algorithms is not analyzed.

3. Definitions, variables and background

This section aims to give the relevant definitions used in the remainder of the paper.

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