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Solving the circular open dimension problem by using separate beams and look-ahead strategies

Hakim Akeb [a], Mhand Hifi [b,*]

[a] ISC Paris School of Management, 22 Boulevard du Fort de Vaux, 75017 Paris, France
[b] Université de Picardie Jules Verne, Equipe ROAD, Unité de Recherche EPROAD, 33 rue Saint-Leu, 80039 Amiens, France

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ABSTRACT

In this paper, a constructive method is investigated for solving the *circular open dimension problem* (CODP), a problem of the Cutting and Packing family. CODP is a combinatorial optimization problem which is characterized by a set of *circular* pieces of known radii and a strip of fixed width W and unlimited length. The objective is to determine the smallest rectangle of dimensions (L, W), where L is the length of the rectangle, that will contain all the pieces such that there is no overlapping between the placed pieces and all the demand constraints are satisfied. The method combines the separate-beams search, look-ahead, and greedy procedures. A study concerning both restarting and look-ahead strategies is undertaken to determine the best tuning for the method. The performance of the method is computationally analyzed on a set of instances taken from the literature and for which optimal solutions are not known. Best-known solutions are obtained.

1. Introduction

Cutting and Packing (C&P) problems have several industrial and commercial applications. A number of industrial and production processes need to pack goods or products efficiently in order to save space during their storage and/or transportation (cf. Baltacioglu et al. [1]). In other cases, industrial machines have to cut material of predetermined shapes from a given two-dimensional plate (cf. Menon and Schrage [2]).

C&P problems have been intensively studied in the literature by applying approximate and exact algorithms (cf. Wäscher et al. [3]). Generally, the objective of a C&P problem consists of cutting or packing a set of items of known dimensions from or into one or more large objects or containers so as to minimize the unused, or waste, portion of the objects. The items and objects may have rectangular, circular, or irregular shapes. These problems are known as difficult to solve exactly and so, heuristics are used for tackling many of them. In this paper, we propose to solve the *circular open dimension problem* (CODP) where the items are circular and the container is a strip. CODP is also known as the strip cutting/packing problem (cf. Akeb and Hifi [4] and Huang et al. [5]).

More precisely, in CODP we are given an initial strip S of fixed width W and unlimited length L, as well as a finite set N of n circular pieces C_i of known radius r_i, $i \in N = \{1, \ldots, n\}$. The objective is to pack (or cut) all the pieces such that (i) the length of the strip S is minimized and (ii) there is no overlapping between pieces, and between pieces and the edges of the strip.

CODP can be formulated as follows:

$$\begin{cases} \min L & & (1)\\ (x_i-x_j)^2+(y_i-y_j)^2 \geq (r_i+r_j)^2,\ j<i,\ (i,j)\in N^2 & & (2)\\ x_i-r_i & \geq 0\ \forall i\in N & (3)\\ y_i-r_i & \geq 0\ \forall i\in N & (4)\\ W-y_i-r_i & \geq 0\ \forall i\in N & (5)\\ L-x_i-r_i & \geq 0\ \forall i\in N & (6)\\ L & \geq \underline{L} & (7) \end{cases}$$

Eq. (1) indicates the value to minimize (the length of the initial strip S). Eq. (2) represents the non-overlap constraint of any pair of separate pieces (C_i, C_j); it means that the distance between the centers of these two circles must be greater than or equal to the sum of their radii. Eqs. (3)–(6) ensure that any piece C_i, $i \in N$, belongs to the target rectangle of dimensions (L, W). Finally, Eq. (7) means that the solution (L) is necessarily greater than or equal to the trivial lower bound $\underline{L} = (\pi \times \sum_{i\in N} r_i^2)/W$.

The remainder of the paper is organized as follows. Section 2 provides a literature review for CODP. Section 3 summarizes the principle of the minimum local-distance position procedure (MLDP). Section 4 describes the BSBIS (Beam Search Binary-Interval Search) algorithm already considered in Akeb and Hifi [4]. Section 5 details the proposed constructive algorithm which

* Corresponding author.
E-mail addresses: hifi@u-picardie.fr, hifi@univ-paris1.fr (M. Hifi).

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combines beam search (using separate beams), multi-start, and look-ahead strategies. In Section 6, the results of the proposed algorithm are evaluated on a set of benchmark instances. A study showing the best tuning of the restarting and look-ahead strategies is also performed in this section. Finally, Section 7 summarizes the contribution of this work and indicates some possibilities for further work.

2. Literature review

Two categories may be distinguished in the circular cutting/packing problems: the container can be a rectangle or a circle. In addition, the circular pieces to be placed can be of equal (identical) or different radii.

Packing identical circles into a circular container is probably the most difficult problem. For example, Graham et al. [6] proposed an approach based on the well-known billiard simulation and the energy function minimization. Recently, López and Beasley [7] developed a heuristic based on the formulation space method for packing equal circles into variously shaped containers (circle, rectangle, triangle, etc.). The problem in this method is expressed by using a non-linear formulation. The method consists of switching between various formulations of the problem (Cartesian and polar coordinates) in order to escape from local optima and/or to improve the solution. This method was also used by Mladenović et al. [8] to solve the problem of packing identical circles into a circle.

The problem of packing variously sized circles into the smallest circle has been studied by several authors. Huang et al. [9] used two algorithms named A1.0 and A1.5; these are based on the maximum hole degree (MHD) heuristic. MHD is based on the use of the circles radii and the distances between the circles. A dynamic adaptive local search was proposed by Hifi and M'Hallah [10]. The method starts with a small radius for the circular container and this radius is then increased at each insertion of a new circular piece. Finally, Akeb et al. [11] proposed a beam search method based on a width-first search in order to place circles of different sizes into the smallest possible circle.

Packing circles into a container of rectangular shape has many industrial and production applications. George et al. [12] proposed a genetic algorithm and a quasi-random approach in order to place unequal circles into a rectangle. A mathematical model was designed by Stoyan and Yaskov [13] to solve the CODP, the method combines a tree search with a reduced-gradient method in order to search for local optima. Hifi and M'Hallah [14] used a genetic algorithm and a constructive procedure for packing circles into a strip. The same problem was studied by Birgin et al. [15] using a non-linear model. Huang et al. [5] proposed two solution procedures for the CODP. The first algorithm is called B1.0 and begins by placing two different circles inside the rectangle (starting configuration) and then calls a greedy procedure so as to place the $n-2$ remaining circles. The first circle in the starting configuration is placed in the bottom-left corner of the rectangle and the second one touches the first circle and one of the edges of the rectangle, or does not touch the first circle but is placed in another corner of the rectangle (so there at most five possible positions for the second circle). The number of possible starting configurations is then bounded by $5m(m-1)$, where m represents the number of different circles (radii) in the instance (or the number of circle types). Note that the greedy procedure may be called $5m(m-1)$ times (the maximum number of starting configurations). The second solution procedure proposed by the authors corresponds to algorithm B1.5 which combines B1.0 with a look-ahead strategy. Kubach et al. [16] proposed a parallel greedy algorithm (denoted by B1.6). B1.6 is based on B1.5 in which a satisfactory tuning of the number of starting configurations is considered. In fact, such an approach focuses on generating more starting configurations than those used in B1.5. Limited computational results showed that such an approach is able to provide four better solutions on the six best ones reached by B1.5. Akeb and Hifi [4] proposed three algorithms for CODP: (i) an open-strip generation solution procedure (OSGSP) based on an optimization problem, (ii) a local beam-search solution procedure that combines beam search and OSGSP, and (iii) a hybrid algorithm that combines beam search, binary interval search, and OSGSP. Finally, Akeb et al. [17] proposed an augmented beam-search based algorithm for the CODP. This algorithm essentially employs a new way of exploring the search tree. Indeed, several beams are initiated, which allows separate searches and a diversification of the search space.

The Beam-search method was previously used by the authors for solving several cutting/packing problems. In [11], a standard beam search method was proposed in order to solve the problem of packing circles into the smallest circle. The algorithm uses a width-first search. In [18], the problem of packing circles into a rectangle of fixed dimensions (a knapsack problem) was studied by using several algorithms based on a standard beam search, restarting strategies, hill-climbing, and look-ahead. An algorithm called SEP-MSBS was proposed by the authors in [17] in order to solve the CODP. The algorithm combines a separate-beams search (a technique proposed by the authors) and a restarting strategy. In this paper, the CODP is solved by using a constructive algorithm; that is an extended and complete version of the method presented in Akeb and Hifi [19]. The principle of the method is based on combining separate-beams search, a multi-start strategy, and a look-ahead. In addition to the work in [19], a study of the parameters of the look-ahead and the multi-start strategies is proposed here. The main objective of the proposed method is to examine how the introduction of the look-ahead strategy could improve existing results even when the execution time limit is fixed to a relatively low value.

3. The minimum local distance strategy (MLDP)

In this section, the principle of the minimum local distance position (MLDP) strategy is described. MLDP is used to evaluate the positions for placing the next circles, and is in this case very important. First, here are the various notations used in the paper:

1. $N=\{1,\dots,n\}$ denotes the set of n circles to be packed,
2. $M=\{1,\dots,m\}$ corresponds to the set of circle types, *i.e.*, the set of different circles (radii) in the instance N ($M \subseteq N$). Note that for the CODP $M \approx N$ meaning that the circles are strongly heterogeneous,
3. The strip, denoted by S, is placed with its bottom-left corner at coordinates (0,0),
4. S_{left}, S_{top}, S_{right}, and S_{bottom} denote the four edges of S,
5. Each circular piece C_i of radius r_i is placed with its center at (x_i,y_i),
6. I_i corresponds to the set of i circles already placed inside the strip S,
7. $\bar{I}_i$ denotes the circles which are not yet placed,
8. P_{I_i} denotes the set of distinct corner positions of the next circle C_{i+1} to be packed given the set I_i,
9. A corner position $p_{i+1} \in P_{I_i}$ for the next circle to place (C_{i+1}) is determined by using two elements a and b. An element is either a piece already placed (*i.e.* $\in I_i$) or one of the three edges of S (S_{left}, S_{top}, S_{bottom}). $T_{p_{i+1}}$ denotes the set composed of both elements a and b, *i.e.*, $|T_{p_{i+1}}|=2$ for each corner position (see Fig. 1).

Fig. 1 shows a configuration in which two circles are already placed ($i=2$) and then $|I_i|=2$. In this case, $I=\{C_1,C_2\}$ where for