Computers & Operations Research 40 (2013) 2103–2115

Contents lists available at SciVerse ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

The traveling purchaser problem, with multiple stacks and deliveries: A branch-and-cut approach

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A R T I C L E I N F O

Available online 26 February 2013

Keywords:
Traveling purchaser problem
Pickup and delivery
Last-in-first-out
Branch-and-cut

A B S T R A C T

The Double Traveling Salesman Problem with Multiple Stacks is a pickup-and-delivery single-vehicle routing problem which performs all pickup operations before the deliveries. The vehicle has a loading space divided into stacks of a fixed height that follows a Last-In-First-Out policy. It has to collect products following a Hamiltonian tour in a pickup region, and then deliver them following a Hamiltonian tour in a delivery region. The aim is to minimize the total routing cost while satisfying the vehicle loading constraints.

A generalization of this problem considers that each product is offered in several pickup locations at different prices, that is, the pickup locations are markets. That means that the pickup tour may not be Hamiltonian, and therefore the set of locations to be visited during the pickup tour is unknown in advance. The delivery locations represent customers, each requiring a product, and all of them must be visited by the vehicle. Thus, this problem has to select a subset of pickup locations to purchase the products, to determine a tour visiting the selected pickup locations, and to design a Hamiltonian tour which visits the delivery locations. The aim is to minimize the purchasing cost plus the total routing cost, subject to the vehicle loading constraints.

This paper introduces and formulates this generalization, called the Traveling Purchaser Problem with Multiple Stacks and Deliveries. It proposes valid inequalities, and adapts some constraints defined for the Double Traveling Salesman Problem with Multiple Stacks by other authors. This formulation motivates a Branch-and-Cut algorithm whose performance has been tested on 240 instances from the literature properly adapted. Our computational experience confirms the effectiveness of the valid inequalities here proposed, and shows that instances of up to 24 products and 32 markets can be solved to optimality.

1. Introduction

The *Double Traveling Salesman Problem with Multiple Stacks* (DTSPMS) is a pickup-and-delivery single-vehicle routing problem which performs the pickup operations before the deliveries, and loads the collected products into a capacitated vehicle as they are picked up. This problem arises when the pickup and delivery regions are widely separated, and the transportation cost between both regions is fixed and therefore not considered as a part of the optimization problem.

The pickup and the delivery points, in addition to a depot in each region, are known in advance. There is a routing cost related to each pair of points for each region. Each product is associated exactly with a pickup point in the pickup region and with a delivery point in the delivery region. All products have identical shape and size, and the vehicle has a loading space divided into stacks of a fixed height. This loading space is big enough to store all products, and the loading operations follow a Last-In-First-Out (LIFO) policy. That means that each loaded product is placed at the top of one of those stacks, and only the products located at the top of a stack can be unloaded from the vehicle.

The DTSPMS collects each product following a Hamiltonian tour in the pickup region starting at the pickup depot, and delivers the products also following a Hamiltonian tour in the delivery region starting at the delivery depot. Note that, the LIFO policy determines the order of collection and delivery, therefore any description of both Hamiltonian tours should specify unambiguously the pickup and the delivery sequence. The aim is to minimize the total routing cost satisfying the stacks height and the LIFO policy. Therefore, the DTSPMS combines two instances of the directed Traveling Salesman Problem (TSP), one for the pickup region and another for the delivery region, with a combinatorial loading problem. This loading problem establishes whether both tours are compatible with respect to the features of the vehicle container.

Pertersen and Madsen [1] introduced the DTSPMS in the context of a real world application on multimodal transport between two separated regions. There has been a growing interest in exact algorithms for the DTSPMS [2,3] and special cases of this problem

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http://dx.doi.org/10.1016/j.cor.2013.02.007

[4] during the last years. Alba Martínez et al. [5] have recently proposed the most successful exact approach presented so far, which is based on a Branch-and-Cut (BC) algorithm. Because of the hardness of the problem we can find some heuristic algorithms in the literature [1,6–8]. The complexity of the loading problem underlying the DTSPMS has been discussed in [9–11], and an approximation scheme has been proposed in [12].

This paper addresses a generalization of the DTSPMS which considers that each product is offered in different locations at different prices. That is, each product may be available at more than one location in the pickup region, and each pickup location may offer more than one product. Furthermore, each product has a purchasing cost that depends on the purchasing point where it is available. To emphasize this aspect each pickup location is henceforth called *market*. The vehicle does not have to visit all markets in this generalization, and therefore the subset of markets forming part of the pickup tour is unknown in advance. This variation of the DTSPMS has to

(i) select a subset of markets to purchase the products,
(ii) determine on the basis of this selection a simple directed cycle which starts at the pickup depot and visits the selected markets,
(iii) design a directed Hamiltonian tour visiting the delivery locations starting from the delivery depot.

The aim is to minimize the purchasing cost plus the total routing cost, subject to the vehicle container constraints. Note that the vehicle container constraints do not refer to the container's capacity, because it is assumed that the container is able to store the full demand. The vehicle container constraints refer to the container organizing policy, which determines the pickup and delivery sequence.

The problem of computing (i) and (ii) is known in the literature as the *Traveling Purchaser Problem* (TPP) [13]. Since this generalization connects an instance of the directed TPP with an instance of the directed TSP by mean of a loading problem, this generalization of the DTSPMS is called the *Traveling Purchaser Problem with Multiple Stacks and Deliveries* (TPPMSD). To our knowledge no previous work has studied this problem.

The TPP considers a set of products to be purchased and a single vehicle originally at a depot. This problem also considers a set of markets, each offering a subset of the product set. There is request of collecting one unit of each product. The purchasing cost of a product depends on the market where it is available. It is also known as the travel cost between each pair of market locations. The TPP aims at selecting a subset of markets and routing them with the vehicle such that all the products are collected and the sum of the purchasing plus the travel cost is minimized. The TPP has been widely studied from the exact [14,15,13,16–18] and the heuristic [21,19,20,22–26] points of view.

Some other problems concern a simple cycle visiting a subset of locations selected with the purpose of collecting a set of items: The *Ring Star Problem* [27] is a variation of the TPP where each product location behaves as a potential market as well. Any product can be purchased in some other markets with an extra assignment cost. This problem has to collect all products minimizing the assignment cost plus the routing cost. The *Covering Tour Problem* [28] is another variation which offers each product at different markets, without any purchasing nor assignment cost. The Covering Tour Problem determines a simple cycle visiting a subset of markets such that all products are collected, minimizing the total routing cost. The *Orienteering Problem* [29] offers exactly one product in each market. Each product has a non-negative prize if it is collected. This problem calls for a simple cycle whose total routing cost does not exceed a given threshold, while visiting a subset of markets with maximum total prize. Note that it does not need to collect all products. Similarly the Orienteering Problem, the *Prize Collecting TSP* [30] offer exactly one product in each location and each product also has a non-negative prize if it is collected. This problem looks for a simple cycle that minimizes the travel cost, subject to a lower bound on the amount of the prize collected.

This paper introduces and formulates the TPPMSD in Section 2. Section 3 describes valid inequalities, and adapts some constraints proposed for the DTSPMS by other authors to the specific features of this problem. Section 4 gives some details on the separation procedures for the valid inequalities introduced in the previous section. The formulation and the new valid inequalities motivate a BC algorithm whose performance has been tested on instances from the literature properly adapted to the TPPMSD. In this context, Section 5 describes a computational experience which studies the effect of the new inequalities proposed here as well as the performance of the BC algorithm. According to this computational experience, the computer implementation is state-of-the-art literature, as it solves a new DTSPMS instance not previously solved by other authors. In addition, it has shown that TPPMSD instances up to 24 products and 32 markets can be solved to optimality. Finally, Section 6 summarizes and comments on the contribution of this paper.

2. Mathematical formulation

This section gives a formal description for the TPPMSD which considers the product collection sequence at each market. This description results in a Linear Integer Programming formulation.

2.1. Definitions

We denote by $I=\{1,\ldots,n\}$ the set of n required products. Let $M=\{1,\ldots,m\}$ be the set of markets offering some products of I. We represent the availability of products at markets by a set of pairs $\Gamma=\{(i,u)\subseteq I\times M \mid$ the product i is available at market $u\}$.

We will also use I to represent the delivery locations since each location requires a unique product. Let us denote by 0^P and by 0^D the depots of the pickup and delivery regions, respectively. We define the TPPMSD on two complete directed graphs, $G^P=(V^P,A^P)$ and $G^D=(V^D,A^D)$, where $V^P=M\cup\{0^P\}$, $V^D=I\cup\{0^D\}$, and A^P and A^D are their respective arc sets without loops.

For each pair $(i,u)\in\Gamma$ we define the purchasing cost d_{iu} as the cost of purchasing the product i at the market u. We also define the routing cost in the pickup region c^P_{uv} for each arc $(u,v)\in A^P$, and in the delivery region c^D_{ij} for each arc $(i,j)\in A^D$.

The vehicle has a loading space, called *container*, organized in r stacks with a capacity for l items each. We denote by R the set of indices identifying each stack, i.e. $R=\{1,\ldots,r\}$. Each stack is only accessible from one of its sides and follows a LIFO policy.

A feasible solution for the TPPMSD may be represented by a tuple (Γ',Q,F,Λ) which consists of

1. an assignment $\Gamma'\subseteq\Gamma$ where each product is assigned to exactly one market,
2. a simple cycle Q which starts at the depot 0^P and visits those markets with assigned products,
3. a Hamiltonian tour F which starts at the depot 0^D visiting all delivery locations I,
4. a loading plan $\Lambda\subseteq I\times R$ which assigns each product to a stack of the container so that both the pickup and the delivery tours are simultaneously compatible with the container configuration.

The TPPMSD aims at selecting a feasible solution (Γ',Q,F,Λ) that minimizes the purchasing cost, computed as $\sum_{(i,u)\in\Gamma'}d_{iu}$, plus the routing costs of Q and F. Note that, in contrast to the DTSPMS, the set of vertices forming part of the pickup tour is unknown