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Case study

Matched filtering method for separating magnetic anomaly using fractal model

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ABSTRACT

Fractal/scaling distribution of magnetization in the crust has found with growing body of evidences from spectral analysis of borehole susceptibility logs and magnetic field data, and fractal properties of magnetic sources have already been considered in processing magnetic data such as the Spector and Grant method for depth determination. In this study, the fractal-based matched filtering method is presented for separating magnetic anomalies caused by fractal sources. We argue the benefits of considering fractal natures of source distribution for data processing in magnetic exploration: the first is that the depth determination can be improved by using multiscaling model to interpret the magnetic data power spectrum; the second is that the matched filtering can be reconstructed by employing the difference in scaling exponent together with the corrected depth and amplitude estimates. In the application of synthetic data obtained from fractal modeling and real aeromagnetic data from the Qikou district of China, the proposed fractal-based matched filtering method obtains more reliable depth estimations as well as improved separation between local anomalies (caused by volcanic rocks) and regional field (crystalline basement) in comparison with the conventional matched filtering method.

1. Introduction

The concept of fractal geometry is introduced by Mandelbrot (1967) to describe, model and analyze the complex phenomenon or process manifesting self-similarity or scale invariance. The past 40 years have seen the extension of fractal concept from geometric sets to multiscaling fields, which significantly increased its applicability (Mandelbrot, 1989; Cheng, 2007; Lovejoy and Schertzer, 2007; Chandrasekhar et al., 2013). A wide range of geofields have been discussed in various power-law scaling or fractal terms, such as radial-density of mineral deposits (Carlson, 1991), density-area of geochemical concentrations (Cheng et al., 1994), magnitude-frequency of earthquakes (Turcotte, 1997) and spectrum-frequency of topography (Gagnon et al., 2006) and geochemical landscapes (Chen et al., 2016), to name but few examples. Such scaling behaviors may represent the end products of numerous independent or nonlinear geo-processes in the lithosphere (Cheng, 2007; Lovejoy and Schertzer, 2007). In last two decades, growing body of evidences from borehole susceptibility logs and magnetic surveys showed that the distribution of crustal magnetization exhibits statistical self-similarity which depicts a power-law dependence of power density spectrum on frequency, the so-called scaling ($1/f^{\beta}$) noise (e.g., Pilkington and Todoeschuck, 1993; Maus and Dimri, 1994; Lovejoy and Schertzer, 2007; Bansal and Dimri, 2014). The scaling exponent (β) measures the correlation of adjacent values within the series. $\beta < 0$ indicates a anti-correlated series; $\beta = 0$ indicates a completely uncorrelated series (e.g., white noise); $\beta > 0$ indicates a correlated series; the series becomes more correlated when β becomes more positive.

In magnetic exploration, the more commonly used assumption in data interpretation is homogeneous source, certainly, standing in contrast to its complex forms observed from well logs. From a inhomogeneous distribution point of view, a random uncorrelated (statistical) model was first used to model magnetization distribution and to interpret magnetic data using the spectral methods (Naidu, 1968; Spector and Grant, 1970). Subsequently, with the fractal nature of sources becoming evident, numerous efforts have been made with an incentive of using fractal concept to facilitate the interpretation of magnetic data. These applications include the kriging interpolation using a fractal covariance model (Pilkington et al., 1994), the inversion for fractal magnetic source distributions (Maus and Dimri, 1995), the Curie depth estimation using scaling spectral analysis (Maus et al., 1997; Ravat et al., 2007;

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Bouligand et al., 2009; Bansal et al., 2011; Bansal et al., 2013), the model-based filtering method (Pilkington and Cowan, 2006) and grid preparation using scaling noise (Pilkington and Keating, 2012). Specifically, fractal model helps to interpret the power spectrum of magnetic data by considering a frequency power-law (scaling) decay besides the depth-dependence exponential decay, and it conducts and improves the depth determination of ensemble source by employing a scaling exponent (β) to correct this power-law decay before applying the Spector and Grant (1970) method.

In this study, we are devoted to use the fractal/multifractal model to facilitate the magnetic field separation of using matched filtering (MF) method. The MF was proposed by Syberg (1972), mainly employing the separation of layers and amplitude ratio to construct the filtering transfer function. However, the conventional MF method is based on the random uncorrelated model, which ignored the fractal/multifractal nature of sources. With this in mind, the proposed fractal-based MF method is centered on two aspects for improving the transfer function: the first is to correct the estimations of depth and amplitude by using scaling spectral analysis. The second is to employ the difference in scaling exponent resulting from fractal/multifractal sources, which usually show different statistical self-similarities in term of inhomogeneous scaling exponent ranging between 1.5 and 5.0 (Bouligand et al., 2009; Pilkington and Keating, 2012). Finally, the fractal-based MF method discussed in this paper is tested using synthetic data generated by fractal modeling and real aeromagnetic data from the Qikou district of China. In this study, we assume that magnetic anomalies are purely caused by induced magnetization without the effects of remanent magnetization.

2. Methodology

2.1. Fractal/scaling nature of sources

Fractals are natural consequence of self-similarity/self-affinity associated with scale-invariance, which refers to the property of a system that does not change by changing scales. This property, in general, can be identified by a power-law relation between a measure $M(\delta)$ and the measuring unit δ, $M(\delta) \propto \delta^{E-D}$, where $\propto$ stands for proportionally. E, D and $E-D$ represent topological dimension, fractal dimension and fractal codimension (scaling exponent), respectively. As mentioned previously, numerous power-law type functions have been used to describe the fractal natures in geosciences. Perhaps the most popular and simplest power-law model to describe fractal geofields (e.g., topography, geochemical landscapes, rains and clouds, etc) is the scaling noise. Recent studies have shown many evidences to support the fractal or at least scaling nature of magnetization distributions by using spectral analysis (e.g., Leonardi and Kümpel, 1996; Zhou and Thybo, 1998; Bansal et al., 2010, etc.), depicting that the power density spectrum (ϕ_m) of magnetization variables has a power-law dependence on the wavenumber (k)

$$\phi_m(k) \propto k^{-\beta_m}. \tag{1}$$

The scaling exponent β_m, as indicator of persistence or type of correlation, could quantify the spatial statistic property of magnetization distributed within the crust. Using a stochastic fractal distribution of 3D magnetization with an isotropic scaling exponent (β_m), Pilkington and Todoeschuck (1993) deduced that the power spectrum (S) of the resulting magnetic field can be written as

$$S(u, v, z) = \frac{8}{\pi}\frac{(-\beta_m - 1)!!}{(-\beta_m)!!}e^{-2kz}k^{-\beta_m+1}, \tag{2}$$

where u and v are the horizontal wavenumber, $k = \sqrt{u^2 + v^2}$ is the radial wavenumber, !! is the double factorial and z is the depth to the top of source distribution. Above model suggests that a fractal magnetic source with β_m could produce fractal magnetic field (at the top of source, i.e. $z=0$) whose power spectrum possesses frequency scaling decay with scaling exponent $\beta_f = \beta_m - 1$.

A review of publish data conducted by Bouligand et al. (2009) showed a wide range of β_m estimates between 1.5 and 5.0 for 3D crustal magnetization, based on observation scale from 1 m to 1000 km. Also, their observations suggest that the range of β_m-*value* differs significantly among igneous ($3.1 \le \beta_m \le 5.0$), metamorphic ($2.4 \le \beta_m \le 4.0$) and sedimentary ($1.5 \le \beta_m \le 3.3$) unit/province. In general, the β_m-*value* depends on heterogeneity within the lithological units (Bansal et al., 2010), and measures the composition and balance of stochastic and deterministic components (Wu et al., 1994). For instance, sedimentary provinces generally show smaller β_m-*value* due to the uncorrelated distributions of magnetizations where stochastic components (e.g., heterogeneity and measurement errors) play a leading role, while igneous provinces exhibit bigger β_m-*value* due to the correlated distribution of magnetizations where deterministic components (e.g., lithological units and regional trend) play a leading role. These facts seem to suggest that crustal magnetizations scale with multiple scaling behaviors, the so-called multifractal/multiscaling. Multifractal is a type of fractal in contrast to the monofractal that shows a homogeneous scaling rule across scales, and the multifractal natures of crustal magnetization have already been reported and argued in numerous literatures (e.g., Fedi, 2003; Lovejoy and Schertzer, 2007; Gettings, 2012).

2.2. Matched filtering method using fractal model

Regional–residual separation is a common issue in the interpretation of magnetic data. The regional usually implies deep-sources effects while residual/local implies shallow effects. Many filtering methods have been designed to implement regional–residual separation, such as matched filtering (MF), wavelet decomposition (Fedi and Quarta, 1998) and empirical model decomposition (Huang et al., 2010), etc. The advantage of MF over other kind of filters is that the MF has geologically constrained benefits including a class of geological models and its depth determination, whereas other methods do not have. Based on a stochastic uncorrelated source distribution, the resulting magnetic field power spectrum is simply characterized by a depth-dependent exponential decay; therefore, the MF method was designed for separating regional–residual components by using the natural break in the spectrum slope (Spector and Grant, 1970; Syberg, 1972). For a simplest case of two ensemble sources, we have a deep-seated source and a shallow source with average depth to the top of the body H and h, respectively. The power spectrum of magnetic field caused by deep (S_1) and shallow (S_2) sources can be written as

$$S_1(k) = A^2 e^{-2kH}, \tag{3}$$

$$S_2(k) = a^2 e^{-2kh}. \tag{4}$$

where A^2 and a^2 are the intercept (amplitude) value of power spectrum.

However, above simplified spectrum model ignored the additional frequency power-law (scaling) decay evidenced in real power spectrum of magnetic data (Pilkington et al., 1994; Maus