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Volterra series truncation and kernel estimation of nonlinear systems in the frequency domain

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ABSTRACT

The Volterra series model is a direct generalisation of the linear convolution integral and is capable of displaying the intrinsic features of a nonlinear system in a simple and easy to apply way. Nonlinear system analysis using Volterra series is normally based on the analysis of its frequency-domain kernels and a truncated description. But the estimation of Volterra kernels and the truncation of Volterra series are coupled with each other. In this paper, a novel complex-valued orthogonal least squares algorithm is developed. The new algorithm provides a powerful tool to determine which terms should be included in the Volterra series expansion and to estimate the kernels and thus solves the two problems all together. The estimated results are compared with those determined using the analytical expressions of the kernels to validate the method. To further evaluate the effectiveness of the method, the physical parameters of the system are also extracted from the measured kernels. Simulation studies demonstrates that the new approach not only can truncate the Volterra series expansion and estimate the kernels of a weakly nonlinear system, but also can indicate the applicability of the Volterra series analysis in a severely nonlinear system case.

1. Introduction

Volterra series [1] have been used for the modelling and analysis of nonlinear systems in many industries such as marine [2], automotive [3], structural [4], biological [5], and communication systems [6]. The Volterra model is a direct generalisation of the linear convolution integral and provides an intuitive system representation. The multidimensional Fourier transform of the Volterra kernels is a natural extension of the linear frequency response function to the nonlinear case and is often referred to as the Generalised Frequency Response Functions (GFRFs). The GFRFs have received much more research interest over the time-domain Volterra kernels. This is because important nonlinear phenomena such as harmonics, intermodulation and gain expansion/depression can easily be explained by the interactions between different frequency components and orders of these GFRFs [7].

The GFRFs of nonlinear systems can be determined by either a parametric-model-based method or a nonparametric-model-based method [8]. In the parametric approach, a nonlinear parametric model is first identified from the input–output data. The GFRFs are then obtained by mapping the resultant model into the frequency domain using the probing method [9].

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The nonparametric approach is often referred to as frequency-domain Volterra system identification and is based on the observation that the Volterra model of nonlinear systems is linear in terms of the unknown Volterra kernels, which, in the frequency domain, corresponds to a linear relation between the output frequency response and linear, quadratic, and higher order GFRFs. This linear relationship allows the use of a least squares (LS) approach to solve for the GFRFs. Several researchers [10–12] have used this method to estimate the GFRFs. But they usually made the assumption that it is known a priori that the system under study can be represented by just two or three terms. However, such information is rarely available a priori.

It is well known that the Volterra series cannot represent severely non-linear systems. And even for a weakly nonlinear system, the order of the Volterra series expansion to achieve an approximation accuracy may still be very high. This indicates that the estimation of the GFRFs is related to the truncation of the Volterra series expansion. And because nonlinear system analysis using Volterra series is usually based on a truncated description, the study on the truncation of the Volterra series expansion is important. Although Billings and Lang [13] proposed an algorithm to truncate Volterra series representations, the algorithm makes an assumption that the GFRFs are known a priori or they can be obtained from the time-domain model, which is, however, not practical in many cases.

In this paper, a novel approach utilising a complex-valued orthogonal least squares (OLS) algorithm regularised by an adjustable prediction error sum of squares (APRESS) criterion will be developed for both the truncation of the Volterra series expansion and the estimation of the GFRFs.

2. Volterra modelling of nonlinear systems in the time and frequency domain

The output $y(t)$ of a single input single output (SISO) analytical system can be expressed as a Volterra functional polynomial of the input $u(t)$ to give

$$y(t) = \sum_{n=1}^{\bar{N}} \mathrm{y}^{(n)}(t) \tag{1}$$

where $\bar{\bar{N}}$ is the maximum order of the system nonlinearity and $y^{(n)}(t)$ is the nth-order output of the system, which is given by

$$y^{(n)}(t) = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} h_n(\tau_1, \cdots, \tau_n) \prod_{i=1}^{n} u(t - \tau_i) d\tau_i,\ n \geq 1 \tag{2}$$

where $h_n(\tau_1,\cdots,\tau_n)$ is a real valued function of $\tau_1,\cdots,\tau_n$ called the nth order impulse response function or Volterra kernel of the system [1]. Volterra generalised the linear convolution concept to deal with nonlinear systems by replacing the single impulse response with a series of multidimensional integration kernels. The nth-order Volterra kernel describes nonlinear interactions among n copies of the input. The multidimensional Fourier transform of the nth-order Volterra kernel yields the nth-order transfer function or generalised frequency response function (GFRF)

$$H_n(\bar{j}\omega_1, \cdots, \bar{j}\omega_n) = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} h_n(\tau_1, \cdots, \tau_n) e^{-\bar{j}(\omega_1\tau_1 + \cdots + \omega_n\tau_n)} d\tau_1 \cdots d\tau_n \tag{3}$$

which is a natural extension of the concept of the linear frequency response function to the nonlinear case. In Eq. (3), $\bar{j}$ is the imaginary unit.

The nth-order kernel and the kernel transform are not unique because an interchange of arguments in $h_n(\tau_1,\cdots,\tau_n)$ may give different kernels without affecting the input–output relationships. To ensure that the GFRFs are unique, they are symmetrised to give

$$H_n^{sym}(\bar{j}\omega_1, \cdots, \bar{j}\omega_n) = \frac{1}{n!} \sum_{\substack{all\ \ permulations \\ of\ \ \{\omega_1,\cdots,\omega_n\}}} H_n^{asym}(\bar{j}\omega_1, \cdots, \bar{j}\omega_n) \tag{4}$$

Using the concept of GFRF, the general relationship between the input spectrum $U(\bar{j}\omega)$ and the output spectrum $Y(\bar{j}\omega)$ can be obtained as

$$Y(\bar{j}\omega) = \sum_{n=1}^{\bar{N}} \frac{1}{\sqrt{n}(2\pi)^{n-1}} \int_{\omega_1 + \cdots + \omega_n = \omega} H_n(\bar{j}\omega_1, \cdots, \bar{j}\omega_n) \prod_{i=1}^{n} U(\bar{j}\omega_i) d\sigma_\omega \tag{5}$$

where $\int_{\omega_1+\cdots+\omega_n=\omega} (\cdot) d\sigma_\omega$ denotes the integration of $(\bullet)$ over the n-dimensional hyperplane $\omega_1+\cdots+\omega_n=\omega$.

When the system is subject to a harmonic input such as

$$u(t) = |A|\cos(\Omega t + \angle A) \tag{6}$$