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Review

Synchronization in complex networks and its application – A survey of recent advances and challenges ☆

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ABSTRACT

Complex networks have, in recent years, brought many innovative impacts to large-scale systems. However, great challenges also come forth due to distinct complex situations and imperative requirements in human life nowadays. This paper attempts to present an overview of recent progress of synchronization of complex dynamical networks and its applications. We focus on robustness of synchronization, controllability and observability of complex networks and synchronization of multiplex networks. Then, we review several applications of synchronization in complex networks, especially in neuroscience and power grids. The present limitations are summarized and future trends are explored and tentatively highlighted.

Contents

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1. Introduction

Synchronization of a number of coupled systems has been widely observed in numerous distinct scenarios such as neuroscience, systems biology, electrochemistry, earth science, social societies and engineering (Arenas, Guilera, Kurths, Moreno, & Zhou, 2008; Dahlem et al., 2013; Gao, Chen, & Lam, 2008; Gu, Pasqualetti, Cieslak, Grafton, & Bassett, 2014; Huang, Ho, & Lu, 2012; Jadbabaie, Lin, & Morse, 2003; Li, Ho, & Lu, 2013; Lu, Kurths, Cao, Mahdavi, & Huang, 2012; Maraun & Kurths, 2005; Pikovsky, Rosenblum, & Kurths, 2001; Ren & Beard, 2008; Saber & Murray, 2004; Wielanda, Sepulchre, & Allgöwer, 2011; Zamora-López, Zhou, & Kurths, 2010). The analysis of synchronization is strengthened due to the fact that natural systems, which we intend to understand and exploit, are often interacted closely from different perspectives, determining the complex dynamics of system's properties. For instance, in Uhlhaas and Singer (2006), it is experimentally verified that synchronization plays an important role in the pathogenesis of several neurological diseases, such as Parkinson's disease, Alzheimer's disease and essential tremor (Arenas et al., 2008). In Machowski, Bialek, and Bumby (2008), Rohden, Sorge, Timme, and Witthaut (2012), power grid networks need to attain synchronization to make the entire smart grid operate in a steady state.

Synchronization is a widely studied topic in physics, while the consensus problem of multi-agent systems is an important research problem in engineering (Bakule, 2014; Lovisari & Zampieri, 2012; Sepulchre, 2012). Mathematically, the definitions for synchronization and consensus are quite similar (Cao, Yu, Ren, & Chen, 2013; Wielanda et al., 2011). The main difference is that synchronization focuses on networks with self-dynamics (linear or nonlinear dynamics) and therefore the final agreement state could be time-varying. Nevertheless, in multi-agent systems, the self-dynamics of each agent is usually neglected and thus the asymptotic consensus state is in general a constant (Cao et al., 2013; Wielanda et al., 2011). Recently, more and more researchers borrow ideas from interdisciplinary areas to study issues they care about of complex networks.

Reviews on the advances made in synchronization of complex networks or coordination of multi-agent systems never cease. Some summaries have been presented with various foci in different phases such as synchronization in complex networks (Arenas et al., 2008), synchronization in complex oscillator networks (Döfler & Bullo, 2014), coordination of multi-agent systems (Cao et al., 2013; Saber, Fax, & Murray, 2007), collective motions (Vicsek & Zafeiris, 2012), regulatory networks (Fiedler, Mochizuki, Kurosawa, & Saito, 2013; Mochizuki, Fiedler, Kurosawa, & Saito, 2013) and oscillation death versus amplitude death (Koseska, Volkov, & Kurths, 2013; Saxena, Prasad, & Ramaswamy, 2012; Zou, Senthilkumar, Zhan, & Kurths, 2013).

During the past decades, extensive studies on synchronization in complex networks have been carried out by both physical and control communities assuming different contexts, and various approaches have been proposed on how to deal with synchronization in complex networks. Many systematic results in this regard have unfolded with respect to the models, the methods and the different approaches for handling synchronization of complex networks. Here, we list some recent important topics in the area of synchronization or related ones:

(1) robustness of synchronization in complex networks;
(2) controllability of complex networks;
(3) observability of complex networks;
(4) synchronization of multiplex networks;
(5) explosive synchronization of complex networks;
(6) chimera states of complex networks;
(7) oscillation death and/or amplitude death of complex networks.

Since explosive synchronization and chimera states do not generally fall into the scope of control-oriented investigations (actually within the scope of statistical physics and nonlinear physics) and some reviews on oscillation death or amplitude death of complex networks have been reported (Koseska et al., 2013; Saxena et al., 2012), the state of art on them will not be pursued here. In this survey, our main focus is on synchronization in complex networks related to both control theory and physics, and review related advances by paying special attention to those which previous surveys did not refer to. Our purpose is to establish a connection between physics and engineering by drawing the attention from both areas to circumvent the above mentioned problems by developing appropriate control theories and approaches.

We try to present a survey on recent important results in synchronization of complex networks here. While covering all the contributions seems to be impossible, we devote ourselves to discussing explicit research lines and helping to categorize problems and methodologies. The survey is organized as follows. In Sections 2.1–2.4, we overview the robustness of synchronization in complex networks, controllability and observability of complex networks and synchronization of multiplex networks, respectively. In particular, the topics of controllability of complex networks are categorized into three classes. In Section 3 we focus on the applications of synchronization in complex networks, ranging from cancer therapy and power grids to neuroscience. Finally, a brief summary and outlook are presented in Section 4.

Basic Notations: In this paper, the concept of "controllability" is based on typical works in complex networks (Liu, Slotine, & Barabási, 2011) and control theory (Kalman, 1963; Rugh, 1996). $l \in [1, N]$ represents the number of driver nodes of a network, where N is the network size. $\delta_{\mathcal{D}}(\cdot)$ denotes the characteristic function of the set $\mathcal{D}$, i.e., $\delta_{\mathcal{D}}(i) = 1$ if $i \in \mathcal{D}$; otherwise, $\delta_{\mathcal{D}}(i) = 0$. Define a graph by $\mathcal{G} = [\mathcal{V}, \mathcal{E}]$, where $\mathcal{V} = \{1, \cdots, N\}$ and $\mathcal{E} = \{e(i,j)\}$ are the vertex set and the edge set, respectively. The graph $\mathcal{G}$ is assumed to be directed, weighted and simple. Let the weighted and directed matrix $L = [l_{ij}]_{i,j=1}^{N}$ be the Laplacian matrix of graph $\mathcal{G}$, which is defined as follows: for any pair $i \neq j, l_{ij} < 0$ if $e(i,j) \in \mathcal{E}$; otherwise, $l_{ij} = 0$. $l_{ii} = -\sum_{j=1, j\neq i}^{N} l_{ij}$ $(i = 1, 2, \cdots, N)$.

2. Main survey

This part is divided into four such parts including robustness in synchronization, controllability of complex networks, observability of complex networks and synchronization of multiplex networks. In discussions for each topic, we shall first make a review on the main achievements and present some limitations of current research.

2.1. Robustness in synchronization

Consider a network of N identical systems governed by the following equation:

$$\dot{x}_i(t) = f(x_i, t) - c\sum_{j=1}^{N} l_{ij} h(x_j(t)), \quad i = 1, \cdots, N, \tag{1}$$

$x_i(t) = [x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t)]^T \in \mathbb{R}^n \quad (i = 1, 2 \cdots, N)$ is the state vector of the ith node; c is the global coupling strength of the network; and $f(x_i, t) = [f_1(x_i, t), \cdots, f_n(x_i, t)]^T$ is a vector function describing the evolution of each individual oscillator in the case of no coupling