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Two inhomogeneities of irregular shape with internal uniform stress fields interacting with a screw dislocation

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A B S T R A C T

Using complex variable methods and conformal mapping techniques, we demonstrate rigorously that two inhomogeneities of irregular shape interacting with a screw dislocation can indeed maintain uniform internal stress distributions. Our analysis indicates that while the internal uniform stresses are independent of the existence of the screw dislocation, the shapes of the two inhomogeneities required to achieve this uniformity depend on the Burgers vector, the location of the screw dislocation, and the size of the inhomogeneities. In addition, we find that this uniformity of the internal stress field is achievable also when the two inhomogeneities interact with an arbitrary number of discrete screw dislocations in the matrix.

1. Introduction

In a series of recent papers, several authors have used various approaches to demonstrate that stress distributions inside multiple non-elliptical elastic inhomogeneities remain uniform when the surrounding elastic matrix is subjected to a uniform loading at infinity (see, for example, [1–5]). It is well known, however, that a common feature of crystalline solids is the existence of dislocations [6] and that plastic deformation in solids is closely related to dislocation dynamics (see, for example, [7–13]). It is therefore of great interest to ask whether the internal stress distributions inside multiple elastic inhomogeneities of irregular shape can maintain uniformity in the presence of a number of discrete or continuously distributed dislocations in the elastic matrix surrounding the inhomogeneities.

In this paper, we take the first step towards addressing this challenging question, by asking whether it is possible to maintain internal uniform stress inside two inhomogeneities of irregular shape when either a single or multiple screw dislocations are present in a matrix subjected to uniform anti-plane shear stresses at infinity. We propose a simple yet efficient method based on complex function theory and conformal mapping techniques to determine the shapes of the two aforementioned inhomogeneities. We emphasize that our method remains valid when an arbitrary number of screw dislocations exist in the matrix.

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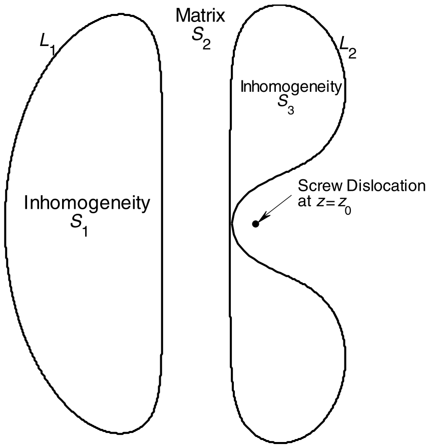

Fig. 1. Two inhomogeneities of irregular shape interacting with a screw dislocation.

2. Two inhomogeneities of irregular shape interacting with a screw dislocation

In the case of anti-plane shear deformations of an isotropic elastic material, the two shear stress components σ_{31} and σ_{32}, the out-of-plane displacement $w = u_3(x_1, x_2)$ and the associated stress function ϕ can be expressed in terms of a single analytic function $f(z)$ of the complex variable $z = x_1 + \mathrm{i}x_2$ as [14]

$$\sigma_{32} + \mathrm{i}\sigma_{31} = \mu f'(z), \quad \mu^{-1}\varphi + \mathrm{i}w = f(z) \tag{1}$$

where μ is the shear modulus of the material. The stresses σ_{31} and σ_{32} are related to the stress function φ through [14]:

$$\sigma_{31} = -\varphi_{,2}, \quad \sigma_{32} = \varphi_{,1} \tag{2}$$

Consider an infinite matrix containing two elastic inhomogeneities of irregular shape. As shown in Fig. 1, let S_1, S_2 and S_3 denote the left inhomogeneity, the matrix and the right inhomogeneity, respectively, all of which are perfectly bonded through the left and the right interfaces L_1 and L_2. The matrix is subjected to a remote uniform anti-plane shear stress field $(\sigma_{31}^{\infty}, \sigma_{32}^{\infty})$ and a single screw dislocation with Burgers vector b_3 located at $z = z_0$. In what follows, the subscripts 1, 2 and 3 (or the superscripts (1), (2) and (3)) are used to identify the associated quantities in S_1, S_2 and S_3, respectively. Our objective is to determine the shapes of the two inhomogeneities which maintain uniform internal stress distributions inside both inhomogeneities.

The continuity conditions of traction and displacement across the two interfaces L_1 and L_2 can be expressed in terms of the corresponding analytic functions in S_1, S_2 and S_3 as follows

$$\begin{aligned} & f_2(z) + \overline{f_2(z)} = \Gamma_1 f_1(z) + \Gamma_1 \overline{f_1(z)} \\ & f_2(z) - \overline{f_2(z)} = f_1(z) - \overline{f_1(z)}, \quad z \in L_1 \end{aligned} \tag{3}$$

$$\begin{aligned} & f_2(z) + \overline{f_2(z)} = \Gamma_3 f_3(z) + \Gamma_3 \overline{f_3(z)} \\ & f_2(z) - \overline{f_2(z)} = f_3(z) - \overline{f_3(z)}, \quad z \in L_2 \end{aligned} \tag{4}$$

where $\Gamma_1 = \mu_1/\mu_2$ and $\Gamma_3 = \mu_3/\mu_2$.

Adding the two conditions in Eq. (3), we obtain

$$f_2(z) = \frac{\Gamma_1 + 1}{2} f_1(z) + \frac{\Gamma_1 - 1}{2} \overline{f_1(z)}, \quad z \in L_1 \tag{5}$$

Similarly, from Eq. (4), we have

$$f_2(z) = \frac{\Gamma_3 + 1}{2} f_3(z) + \frac{\Gamma_3 - 1}{2} \overline{f_3(z)}, \quad z \in L_2 \tag{6}$$

We now construct the following conformal mapping function for the matrix:

$$\begin{aligned} & z = \omega(\xi) = R\left[\frac{1}{\xi - \lambda} + \frac{p}{\xi - \lambda^{-1}} + \frac{\Lambda^{-1}p}{\rho\xi - \lambda^{-1}} + q\log\frac{\xi - \bar{\xi}_0^{-1}}{\xi - \lambda^{-1}} + \Lambda^{-1}q\log\frac{\rho\xi - \bar{\xi}_0^{-1}}{\rho\xi - \lambda^{-1}} + \sum_{n=1}^{+\infty}\left(a_n\xi^n + a_{-n}\xi^{-n}\right)\right] \\ & \xi(z) = \omega^{-1}(z), \quad 1 \le |\xi| \le \rho^{-\frac{1}{2}} \end{aligned} \tag{7}$$