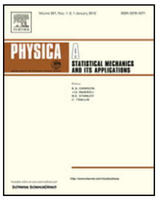
Physica A 392 (2013) 4373–4379

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa



How much information is needed to be the majority during the binary-state opinion formation?

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HIGHLIGHTS

- To propose the nonlinear majority-rule model with the conservation law.
- To find the bifurcation phenomena in the opinion formation.
- To show the scaling relationship between the bifurcation point β_2 and the mean connectivity $\langle k \rangle$.

ARTICLE INFO

Article history:
Received 15 November 2012
Received in revised form 26 March 2013
Available online 25 May 2013

Keywords:
Opinion formation
Bifurcation phenomena
Scale-free network
The conservation law

ABSTRACT

In this paper, we try to propose a toy model, which follows the majority rule with the Fermi function, to uncover the role of the heterogeneous interaction between individuals in opinion formation. In order to do this, we define the impact factor IF_i, says individual i, as the exponential function of its connectivity k_i with the tunable parameter β. β also shows the public information that can be collected by individuals in the system. We realize our model in scale-free networks with mean connectivity $\langle k \rangle$. We find that much more public information ($\beta > \beta_2$) and less public information ($\beta < \beta_1$) cannot let either of the two opinions be the majority during the opinion formation. Furthermore, β_1 is a constant and equal to $-0.76(\pm 0.04)$, and β_2 decreases as a power-law function of the mean connectivity $\langle k \rangle$ of the network. Our work can provide some perspectives and tools to understand the diversity of opinion in social networks.

1. Introduction

Recently, the study of complex networks and human behavior has attracted an increasing interest among physicists. The main reason is that human behavior is profoundly affected by the influenceability of individuals and the social networks that link them together [1]. It has been realized that many real social networks arising in society, such as collaborations between actors [2,3] and scientists [4,5], web-based social networks [6], peer-to-peer social networks [7], and BBS networks [8], share some universal characteristics, such as the small-world effect and the power low degree distribution with the exponent $\gamma (2 < \gamma < 3)$. The primary question is how those features affect human behavior, especially spreading phenomena, such as disease epidemiology and opinion formation? Here, we address opinion formation in a scale-free network to show the role of the heterogeneous interaction between individuals.

In order to do this, several models have been studied recently, such as the Sznajd model [9], the voter model [10,11], the majority-rule model [12] and the Deffuant model [13]. Sood and Redner studied the voter model on a scale-free network with different exponents of the connectivity distribution and found that the mean time T_N (measuring the time when the

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http://dx.doi.org/10.1016/j.physa.2013.05.032

system reaches consensus state) scales as $N^{(2\gamma-4)/(\gamma-1)}$ [14]. Lambiotte studied the effects of degree heterogeneity on the majority-rule model and found that the location of the order–disorder transition depends on the exponent γ and a non-equipartition state exists between two types of individuals [15]. Then, Guan et al. studied the effects of heterogeneous influence among agents on the majority-rule model and found that the location of the order–disorder transition depends on the heterogeneous influence probability w among different types of individuals [16]. In these works, the consideration of heterogeneity is the simple networks with different types of individuals and the same influence between individuals. However, the individuals around us are diverse and the influence between individuals does depend on the properties of the two who interact with each other. The obvious question is how to measure the influence of each individual? One natural assumption is that the influence of an individual is related to his/her connectivity (i.e., vertices to which he/she is linked) in the opinion cycle. For example, J.O. Indekeu assumed that the influence of an individual is inversely proportional to his/her connectivity [17]. Another example is that the influence of a famous star (i.e., more fans to whom he/she is linked) on some field is much stronger than that of a common person. Hence, we introduce the tunable parameter β to describe the influence of an individual's connectivity on opinion formation.

In this paper, we propose a toy model [18] of opinion formation following the majority rule with the overturned Fermi function, because each person will change his/her opinion following the principle of herding behavior to pursue the maximal profit according to psychology [19]. Then, we realize our model in scale-free networks with mean connectivity $\langle k\rangle$, which affects the order–disorder transition point of the variant of majority-rule [20]. We find that the critical order parameter m_* undergoes a bifurcation phenomenon as β increases. Either of the two opinions will be the majority (i.e., more than half of individuals share the same opinion and $|m_*| > 0$) when β falls in the range of $[\beta_1, \beta_2]$, where β_1 and β_2 are bifurcation points that are totally independent of the size of the network, but β_2 is related to the mean connectivity $\langle k\rangle$.

2. Description of our model and its realization

Consider a network of N vertices, representing individuals, joined in pairs by E edges, representing acquaintance or collaboration between individuals. Here, we consider the loop (i.e., the self-connectivity), which reflects the self-feedback of an individual. On some topic of interest, individual i is assumed to hold one of the two possible opinions, denoted by $o_i \in \{+1, -1\}$. During the interaction between individuals, each individual possesses his/her impact factor on his/her local neighbors. In our daily life, the impact factor reflects the quantity of information contributed to his/her local neighbors and the authority. Hence, the impact factor is also called public information [21], denoted by IF_i, which can be collected by others and is defined as an exponential function of its degree k_i as follows:

$$IF_i = k_i^{\beta}, \tag{1}$$

where β is the tunable parameter that reflects the quantity of public information contributed according to the topology properties of individuals in the system. If $\beta = 0$, all individuals have the same impact factor on their local neighbors. Our model reduces to an improved majority-rule model with a flip-probability. If β is positive, an individual with much more connectivity has a stronger impact factor to influence his/her local neighbors and contribute much more public information. If β is negative, the impact factor of the individual is inversely proportional to his/her connectivity [17]. When β tends to $-\infty$, the impact factor k^{β} tends to zero, and individuals can be considered as isolated and choose one of the two opposite opinions with the same probability 1/2.

At each time step, each individual is chosen as the active one with probability P_{act}. Take an active individual i for example, he/she collects the public information from his/her local neighbors, say

$$\begin{cases} E_i^s = \sum_j k_j^{\beta} A_{ij} \delta_{o_j,o_i}; \\ E_i^d = \sum_j k_j^{\beta} A_{ij} (1-\delta_{o_j,o_i}), \end{cases} \tag{2}$$

where $A = \{A_{ij}\}$ is the adjacency matrix and $A_{ij} = 1$ when an edge between individuals i and j exists, $A_{ij} = 0$ otherwise. Specially, $A_{ii} = 1$ shows the self-influence of individual i, since each individual is assumed to have his/her intrinsic property to keep his/her current opinion without changing. The δ function is defined as

$$\delta_{o_i,o_j} = \begin{cases} 1, & o_i = o_j, \\ 0, & o_i \neq o_j. \end{cases} \tag{3}$$

Hence, E_i^s (E_i^d) is the public information that individual i collected from his/her local neighbors with the same (opposite) opinion. Then, the total local public information collected by individual i is

$$E_{i_{tot}} = E_i^s + E_i^d = \sum_j A_{ij} k_j^{\beta}, \tag{4}$$

and the global public information of the whole system is

$$E_{tot} = \sum_{i=1}^{N} E_{i_{tot}} = \sum_{i,j} A_{ij} k_j^{\beta} = \sum_{i=1}^{N} k_i^{\beta}(k_i + 1). \tag{5}$$