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Non-extensive analysis of the seismic activity involving the 2011 volcanic eruption in El Hierro

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ABSTRACT

This paper is focused on the assessment of possible relational elements of the submarine volcanic eruption occurred during 2011 in El Hierro (Canary Islands, Spain) with both the spatio-temporal evolutionary behaviour and the scaling properties of seismic activity before, during and after it. We adopt the non-extensive frequency–magnitude distribution (FMD) model (Silva et al., 2006) for fitting to observed data, where we introduce a weighting function based on the relation between magnitude and energy. Moreover, extension of dependence coefficients in terms of Tsallis entropy (Furuichi, 2006) to the multifractal domain is proposed to study dimensional interaction in terms of the Tsallis generalized dimensions formulation introduced in Angulo and Esquivel (2014). The study shows significant changes in the spatio-temporal dynamics of the seismic activity previous to the volcanic eruption. The combination of these techniques can be seen as a suitable tool in the continuous monitoring of volcanic activity.

1. Introduction

Gutenberg–Richter law (Gutenberg and Richter, 1944), one of the most used tools in the study of earthquakes, expresses the number of events with magnitude above a given threshold. Difficulties of this model for representing accurately events with small and high magnitudes have commonly been attributed to lack of sensitivity of the registration devices. Sotolongo-Costa and Posadas (2004)

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asserted that the frequency–magnitude distribution (FMD) for small events can be explained by the material between the planes of fault, and formulated a new energy distribution model of earthquakes based on the non-extensive statistics (see Tsallis, 1988, 2009). This model was re-analysed by Silva et al. (2006), and later refined by Telesca and Chen (2010). Recent studies have shown successful results describing diverse seismic catalogues using these models, such as Celikoglu and Tirnakli (2012), Sotolongo-Costa (2012), Telesca (2010a,b, 2011, 2012), Telesca and Chen (2010), and Vallianatos et al. (2013). The choice of the estimation method has a significant importance to carry out a correct analysis of the seismic series. As we introduce in Section 2, here we propose the use of a weighted estimation approach based on the relation between magnitude and energy for fitting the non-extensive FMD to de data.

On the other hand, complexity related to scaling behaviour and long-range interactions is present in the spatio-temporal distribution of earthquakes (Bak et al., 2002; Lennartz et al., 2011; Main, 1996; Rundle et al., 2003; Saleur et al., 1996; Turcotte, 1997; Varotsos et al., 2005). State distributions associated with scaling behaviour are formally described in terms of multifractal measures, whereas systems with long-range interactions can be appropriately characterized using the non-extensive statistics (Michas et al., 2013). In particular, Section 3 deals with the study of interaction between components in a system. Furuichi (2006) introduces 'correlation coefficients' in terms of Tsallis entropy to quantify the degree of dependence between components. Scaling behaviour present in complex systems led us to formulate in Angulo and Esquivel (2014) a dependence coefficient in the multifractal domain based on Rényi generalized dimensions. In this paper, we extend this approach in terms of the Tsallis generalized dimensions formulation also introduced in Angulo and Esquivel (2014).

In Section 4, we apply the above tools to study a seismic series registered in El Hierro (Canary Islands, Spain) involving the submarine volcanic eruption occurred during 2011. The main objective is the assessment of possible relational elements of the volcanic eruption with both the spatio-temporal evolutionary behaviour and the scaling properties of seismic activity before, during and after it. Firstly, we assess the spatio-temporal dynamics of the series fitting the FMD of earthquakes model to different data segments derived from specific methodologies focused on the temporal, spatial and spatio-temporal analysis. Secondly, we study the degree of association between the locations of the events and their respective magnitudes using the multifractal Tsallis correlation coefficients proposed. In Section 5, we summarize the methodology and tools proposed, and discuss the main aspects and related interpretations derived in connection with the objectives.

2. Non-extensive FMD of earthquakes

As mentioned above, the Gutenberg–Richter law has been long used as a tool to describe the frequency of earthquakes; however, it is well known that it presents misfit problems for the smallest magnitudes, until it reaches a minimum magnitude threshold. On the other hand, the size distribution of faults and the materials between the planes are important factors to study the seismicity of an area (Sornette, 1999). Sotolongo-Costa and Posadas (2004) suggested that the frequency–magnitude distribution before arriving to this threshold can be described by the fragments between the planes of fault and introduced a non-extensive FMD of earthquakes derived from the fragment–asperity interaction model introduced by Sotolongo-Costa et al. (2000). Silva et al. (2006) proposed a re-analysed version of this model considering both, a revised definition of the q-expectation value (see Abe and Bagci, 2005) and the proportionality $\varepsilon \sim r^3$ (Lay and Wallace, 1995) between released energy and the size of the fragments, obtaining the following expression:

$$\log\left(\frac{N(m > m_{th})}{N}\right) = \left(\frac{2-q}{1-q}\right)\log\left\{1 - \left(\frac{1-q}{2-q}\right)\left(\frac{10^{2m_{th}}}{a^{2/3}}\right)\right\}. \tag{1}$$

This model was refined by Telesca (2011) by considering a derivation of the FMD based on the relation between magnitude and energy $m = \frac{2}{3}\log\varepsilon$ instead $m = \frac{1}{3}\log\varepsilon$:

$$\log\left(\frac{N(m > m_{th})}{N}\right) = \left(\frac{2-q}{1-q}\right)\log\left\{1 - \left(\frac{1-q}{2-q}\right)\left(\frac{10^{m_{th}}}{a^{2/3}}\right)\right\}. \tag{2}$$