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Half spectral composite likelihood approach for estimating spatial–temporal covariance functions

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ABSTRACT

In this paper we propose a method called *half spectral composite likelihood* for the estimation of spatial–temporal covariance functions which involves a spectral approach in time and a covariance function in space. It facilitates the analysis of spectral density of all possible pairwise contrasts at different spatial sites. The proposed approach requires no matrix inversions and the estimators are shown to be consistent and asymptotically normal under increasing domain asymptotic in a fashion similar to Bevilacqua et al. (2012). A simulation study is carried out to assess the performance of the proposed estimation method from statistical and computational viewpoint with respect to difference composite likelihood. The half spectral composite likelihood estimates show better performance with respect to the difference composite likelihood. A real example is given using the Irish wind speed data.

1. Introduction

Spatial–temporal (ST) data are frequently encountered in many scientific disciplines, especially in environmental, meteorological or geophysical contexts. One major challenge in modeling ST data is the high dimensionality of such data. This is crucial because many statistical inferences such as maximum likelihood (ML) estimation and best linear unbiased prediction e.g. kriging (Cressie,

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1993), with massive datasets, involve large dimensional covariance matrices which must be inverted. Therefore, approximating the likelihood functions is a fundamental key to tackle such problems.

There have been several computational efficient approaches to overcome this large dimensional matrix problem, such as assuming cyclic stationary process (Brillinger, 2001), assuming separability on ST covariance functions (Genton, 2007), tapering the covariance matrix (Kaufman et al., 2008), modeling the realizations by a latent process with reduced dimension (Cressie and Johannesson, 2008) and approximating the random field (RF) with a Gaussian Markov RF or basis expansions (Lindgren et al., 2011).

One way to approximate the likelihood without involving the calculation of determinants and inverses is based on Whittle (1954) approximation to the Gaussian negative log-likelihood, which can only be used for datasets observed on a regular lattice. For irregularly spaced Gaussian data, Fuentes (2007) developed a version of Whittle's approximation to the Gaussian negative log-likelihood by introducing a lattice process and truncating the spectral representation of the lattice process. Another spectral method presented by Matsuda and Yajima (2009) extends the definition of a classic periodogram for time series to the irregularly spaced process.

Composite likelihood (CL) as proposed in Lindsay (1988) is also convenient in the setting where the full likelihood is not feasible or is difficult to construct. The idea of CL method is primarily to break a computationally infeasible likelihood into several computationally feasible ones. To construct a CL, one starts with a set of conditional or marginal events for which one can write log-likelihoods easily. The log-composite likelihood is then can be written by adding together the log likelihood of these marginal, bivariate or conditional events while the components are not necessarily independent. Varin (2008) provided a survey of CL applications. Example of CL applications includes Liang and Zeger (1986) for longitudinal data, Lele and Taper (2002) for multivariate normal, and Kuk (2007) for clustered data.

Earlier, Besag (1975) considered a similar approach to CL for spatial data. Heagerty and Lele (1998) and Curriero and Lele (1999) applied CL for binary spatial data. Lele (1997) introduced difference CL methods for the estimation of purely spatial semi-variogram parameters. Bevilacqua et al. (2012) modified this approach to a ST framework and proved that the associated estimator is consistent and asymptotically normal under increasing domain asymptotic. In this paper, our objective is to consider the estimation of the parameters of ST covariance functions using a half-spectral composite likelihood (HSCL) approach which involves a spectral approach in time but a covariance approach in space. The idea of the proposed approach is to facilitate the analysis of spectral density of contrast components at different spatial sites. The attraction of this methods is that the spectral density function is usually very easy and fast to evaluate by fast Fourier transformations. As we will show later in this paper it is the only part of the likelihood function which changes when a new estimate of $\boldsymbol{\theta}$ is produced in an iterative optimization procedure, and therefore the calculations involved in such a procedure can be carried out quite rapidly. Let $Z(\boldsymbol{s}, t)$ be a real-valued stationary ST process defined on $\mathbb{R}^d \times \mathbb{R}$, where $\boldsymbol{s} \in \mathbb{R}^d$ represents the spatial site in d dimension, and $t \in \mathbb{R}$ represents the time. Assume that the second moments for the RF exist and are finite. Write the ST process in the general form

$$Z(\boldsymbol{s}, t) = \iint e^{i(\boldsymbol{s}'\boldsymbol{\omega}+t\tau)} d\boldsymbol{W}(\boldsymbol{\omega}, \tau) = \int e^{it\tau} J(\boldsymbol{s}, \tau) d\tau, \quad (\boldsymbol{s}, t) \in \mathbb{R}^d \times \mathbb{R}$$

where $J(\boldsymbol{s}, \tau) = \int_{\boldsymbol{\omega}} e^{i\boldsymbol{s}'\boldsymbol{\omega}} d\boldsymbol{W}(\boldsymbol{\omega}, \tau)$, $(\boldsymbol{\omega}, \tau) \in \mathbb{R}^d \times \mathbb{R}$ and $\boldsymbol{W}$ is a generalized random function with complex symmetry. Note that $d\boldsymbol{W}(\boldsymbol{\omega}, \tau) = d\boldsymbol{W}^*(-\boldsymbol{\omega}, -\tau)$ ensures that $Z(\boldsymbol{s}, t)$ is real-valued. Suppose that C is a continuous and symmetric function on $\mathbb{R}^d \times \mathbb{R}$. Then C is a covariance function if and only if it is of the form

$$C(\boldsymbol{h}, u) = \iint e^{i(\boldsymbol{h}'\boldsymbol{\omega}+u\tau)} d\boldsymbol{F}(\boldsymbol{\omega}, \tau) = \int e^{iu\tau} H(\boldsymbol{h}, \tau) d\tau, \quad (\boldsymbol{h}, u) \in \mathbb{R}^d \times \mathbb{R} \tag{1}$$

where $H(\boldsymbol{h}, \tau) = \int_{\boldsymbol{\omega}} e^{i\boldsymbol{h}'\boldsymbol{\omega}} d\boldsymbol{F}(\boldsymbol{\omega}, \tau)$ and $\boldsymbol{F}$ is a finite, non-negative and symmetric spectral measure on $\mathbb{R}^d \times \mathbb{R}$. Note that $E|d\boldsymbol{W}(\boldsymbol{\omega}, \tau)|^2 = d\boldsymbol{F}(\boldsymbol{\omega}, \tau)$. We shall call H a "half spectral function" since it depends on the spatial lag $\boldsymbol{h}$ and the temporal frequency τ. If $C(\boldsymbol{h}, u)$ is integrable, the spectral measure is absolutely continuous with a spectral density function $f(\boldsymbol{\omega}, \tau)$. If the spectral density exists, (1)