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# A generalized convolution model and estimation for non-stationary random functions

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ABSTRACT

In this paper, a new model for second order non-stationary random functions as a convolution of an orthogonal random measure with a spatially varying random weighting function is introduced. The proposed model is a generalization of the classical convolution model where a non-random weighting function is considered. For a suitable choice of the random weighting functions family, this model allows to easily retrieve classes of closed-form non-stationary covariance functions with locally varying geometric anisotropy existing in the literature. This offers a clarification of the link between these latter and a convolution representation, thereby allowing a better understanding and interpretation of their parameters. Under a single realization and a local stationarity framework, a parameter estimation procedure of these classes of explicit non-stationary covariance functions is developed. From a local stationary variogram kernel estimator, a weighted local least-squares method in combination with a kernel smoothing method is used to estimate efficiently the parameters. The proposed estimation method is applied on soil and rainfall datasets. It emerges that this non-stationary method outperforms the traditional stationary method, according to several criteria. Beyond the spatial predictions, we also show how conditional simulations can be carried out in this non-stationary framework.



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## 1. Introduction

Estimation and modeling of the underlying spatial dependence structure of data are key elements for kriging and conditional simulations. Simplifying assumptions are often made on the spatial dependence structure. They include the stationarity assumption where the second order association between location pairs is assumed to depend only on the vector between these locations. However, it has become increasingly clear that this assumption is driven more by mathematical convenience than by reality. In practice, it may be doubtful due to many factors, including specific landscape and topographic features of the study domain or other localized effects. These local influences can be reflected by computing local stationary variograms whose characteristics may vary across the study domain. In such cases, a stationary approach may be not suitable because it could provide less accurate predictions, including an incorrect assessment of the prediction error.

Various approaches have been developed over the years, to deal with non-stationarity through second order moments (Guttorp and Schmidt, 2013; Sampson et al., 2001; Guttorp and Sampson, 1994). One of the most popular methods of introducing second order non-stationarity is the convolution approach developed by Higdon (1998) and Higdon et al. (1999). It involves taking a spatial white noise, then averaging it using weights that vary spatially to thereby obtain a second order non-stationary random function. In this way, the resulting spatial dependence structure is allowed to vary across the domain of interest. Higdon (1998) and Higdon et al. (1999) use a spatially varying Gaussian kernel function to induce a non-stationary covariance function. This latter has a closed-form with locally varying geometric anisotropy but is infinitely differentiable which may not be desirable for modeling real phenomena (Stein, 1999). Zhu and Wu (2010) choose a family of spatially varying modified Bessel kernel functions to produce a non-stationary covariance function with local smoothness characteristics that are similar to the Matérn class of stationary covariance functions. One limitation of this approach is that the resulting non-stationary covariance function does not have a closed-form and, in general, can only be evaluated using numerical integration. Moreover, this approach does not take into account the locally varying geometric anisotropy.

Furthermore, a class of explicit non-stationary covariance functions with locally varying geometric anisotropy have been developed by Paciorek and Schervish (2006) and Stein (unpublished report). However, all these classes of analytical non-stationary covariance functions with locally varying geometric anisotropy do not directly derived from a random function model like the convolution representation or the spectral representation. Thus, this does not facilitate their understanding and particularly the interpretation of their parameters. It is useful to have a constructive approach for random functions admitting such closed-form non-stationary covariance functions with locally varying geometric anisotropy. Moreover, the estimation of parameters that govern these latter remains a critical problem. Paciorek and Schervish (2006) enumerate some difficulties and suggest some possible methods including the moving windows approach based on the variogram or the likelihood. Anderes and Stein (2011) mention two typical problems arising with moving windows method. Firstly, the range of validity of a stationary approximation can be too small to contain enough local data to estimate reliably the local spatial dependence structure. Secondly, it can produce non-smooth parameter estimates, leading to discontinuities on the kriging map which is undesirable in many applications. Anderes and Stein (2011) propose a weighted local likelihood approach where the influence of faraway observations is smoothly down-weighted. The drawbacks related to this approach are the computational burden of inverting covariance matrices at every location for parameter estimation and the Gaussian distributional assumption for analytical tractability.

In this work, we are interested primarily in establishing a link between existing classes of explicit non-stationary covariance functions with locally varying geometric anisotropy and a convolution representation. To achieve that, we introduce a new model for second order non-stationary random functions as a convolution of an orthogonal random measure, with a spatially varying stochastic weighting function. This is an extension of the common convolution model where a deterministic weighting function is used. This construction bears some resemblance with the moving average model with stochastic weighting function introduced by Matérn (1986), in order to build some isotropic stationary covariance functions like Matérn and Cauchy families. From this modeling approach, we easily retrieve classes of closed-form non-stationary covariance functions with locally varying